

TECHNOLOGY-SUPPORTED MATH INSTRUCTION FOR STUDENTS WITH DISABILITIES:

TWO DECADES OF RESEARCH AND DEVELOPMENT

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INTRODUCTION

Most people would agree that a major goal of schooling should be the development of students' understanding of basic mathematical concepts and procedures. All students, including those with disabilities and those at risk of school failure, need to acquire the knowledge and skills that will enable them to "figure out" math-related problems that they encounter daily at home and in future work situations. Unfortunately, there is considerable evidence to indicate that this objective is not being met, especially for children exhibiting learning difficulties. Since the first discouraging results of mathematics achievement reported by the National Assessment of Educational Progress (NAEP) in 1973, there has been little evidence to suggest that mathematics achievement has improved significantly, especially for students with disabilities.

According to the NAEP, the majority of elementary and middle school students are not proficient in math. Only 32% of fourth graders and 29% of eighth graders scored at or above the proficient level in math, and unfortunately this is an improvement over previous years (National Center for Education Statistics [NCES], 2003). Students performing at the lowest level on the NAEP assessment are still not achieving the most basic level of math skills, and the gap between low and higher performers persists (NCES, 2003). This gap increases with each year as students with disabilities continue to fall further behind their peers (Cawley, Parmar, Yan, & Miller, 1998). For schools to close this achievement gap and meet the federal guidelines set forth by the No Child Left Behind (NCLB) Act, they must see that *all* students achieve academic proficiency. Technology-based innovations can form the basis of effective approaches to help students who have difficulty with math strive to achieve parity with their peers.

In this paper, we have adopted the term "math difficulty" to include terms frequently used to identify students who have difficulty with mathematics. We have established this term in order to review literature that addresses math achievement for various groups such as students at risk or disadvantage, with dyscalculia, learning disabilities, and so on in order to see a more complete picture of how students struggle with mathematical knowledge and learning.

MATHEMATICAL KNOWLEDGE AND LEARNING

To better understand how to enhance mathematical thinking and learning in today's students, especially students with math difficulty, we must first understand the nature of

mathematical knowledge. Mathematicians and cognitive scientists appear to agree that at least three basic types of mathematical knowledge exist and are required for the development of mathematical literacy and competence. These three types of knowledge are declarative, procedural, and conceptual. A brief overview of these knowledge types is provided below. For a more detailed discussion of this framework, please see Goldman and Hasselbring (1997).

Declarative knowledge can be considered factual knowledge about mathematics. Examples of this type of knowledge are 4 + 7 = 11 or the definition of a square as a four-sided polygon having equal-length sides meeting at right angles. Declarative knowledge serves as the building blocks for procedural knowledge. Procedural knowledge can be defined as the rules, algorithms, or procedures used to solve mathematical tasks. For example, the order of operations is a rule for simplifying expressions that have more than one operation. In contrast, conceptual knowledge goes beyond mere knowledge of discrete facts and procedural steps to a full understanding of interrelated pieces of information. It can be thought of as a connected web of information in which the linking relationships are as important as the pieces of discrete information that are linked. For example, procedural knowledge that is linked to conceptual knowledge can help students select the appropriate mathematical operation to use in a particular situation, because the conceptual knowledge helps them understand the underlying reasons for selecting that operation. Mathematical competency requires the development of an interactive relationship between declarative, procedural, and conceptual knowledge. The development of relationships between these knowledge types is critical for knowledge to be accessible and usable.

A variety of technologies are available to enhance students' mathematical competency by building their declarative, procedural, and conceptual knowledge. The remainder of this paper will review these technologies. This review will be guided by the NCTI Mathematics Matrix found at http://www.citeducation.org/mathmatrix. The matrix identifies six purposes of technology use for supporting student mathematical learning, including (1) building computational fluency; (2) converting symbols, notations, and text; (3) building conceptual understanding; (4) making calculations and creating mathematical representations; (5) organizing ideas; and (6) building problem solving and reasoning. These six purposes support the development of students' declarative, procedural, and conceptual knowledge. Declarative knowledge is developed through technologies that help build computational fluency. Challenges

with procedural knowledge are surmounted with the assistance of technologies that help with converting mathematical symbols and notations, calculating mathematical operations, and inputting/organizing data. Finally, conceptual knowledge is enhanced by technologies designed to build conceptual understanding, problem solving, and reasoning. Research on the use of these different purposes of technologies is reviewed in the next sections.

Building Computational Fluency

The research on computational fluency suggests that the ability to fluently recall the answers to basic math facts is a necessary condition for attaining higher-order math skills. The rationale for this thinking is that all human beings have a limited information-processing capacity. That is, an individual simply cannot attend to too many things at once. Grover Whitehurst, the Director of the Institute for Educational Sciences (IES), noted this research during the launch of the federal Math Summit (2003):

Cognitive psychologists have discovered that humans have fixed limits on the attention and memory that can be used to solve problems. One way around these limits is to have certain components of a task become so routine and over-learned that they become automatic.

The implication for mathematics is that some of the sub-processes, particularly basic facts, need to be developed to the point that they are done fluently and automatically. If this fluent retrieval does not develop, then the development of higher-order mathematics skills—such as multiple-digit addition and subtraction, long division, and fractions—may be severely impaired (Resnick, 1983). Indeed, studies have found that lack of math fact retrieval can impede participation in math class discussions (Woodward & Baxter, 1997), successful mathematics problem-solving (Pellegrino & Goldman, 1987), and even the development of everyday life skills (Loveless, 2003). And rapid math-fact retrieval has been shown to be a strong predictor of performance on mathematics achievement tests (Royer, Tronsky, Chan, Jackson, & Merchant, 1999).

While the research cited above highlights the importance of math fact fluency, the computation capabilities of American students might well be diminishing. Tom Loveless of the Brookings Institute has reviewed responses to select items on the NAEP and concluded that performance on basic arithmetic facts declined in the 1990s (2003). More emphasis needs to be placed on developing rapid, effortless, and errorless recall of basic math facts.

NORMAL DEVELOPMENT OF FLUENT MATH FACTS

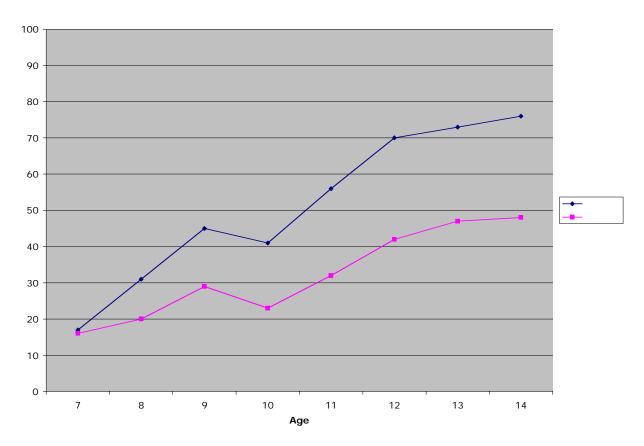
Given the importance of the fluent recall of basic facts, the main concern is how this ability develops. For many children, at any point in time from preschool through at least the fourth grade, they will have some facts that can be retrieved from memory automatically and some that need to be calculated using some counting strategy. From the fourth grade through adulthood, answers to basic math facts are recalled from memory with a continued strengthening of relationships between problems and answers that results in further increases in fluency (Ashcraft, 1985).

The acquisition of math facts in most normally developing children generally progresses from a deliberate, procedural, and error-prone calculation to one that is fast, efficient, and accurate (Ashcraft, 1992; Fuson, 1982, 1988; Siegler, 1988). In contrast, most students with math difficulty, along with those lacking consistent math fact instruction, show a serious problem with respect to the retrieval of elementary number facts. Hasselbring, Goin, and Bransford (1988) found that students with math difficulty are substantially less proficient than their non-math-difficulty peers in retrieving the answers to basic math facts in addition and subtraction. Although information is still emerging about the particular difficulties experienced by these children in the retrieval of this information, the evidence that does exist suggests that these children do not suffer from a conceptual deficit (Russell & Ginsburg, 1984) but rather from some sort of disruption to normal development. What this suggests is that there are huge differences in the amount of instruction individual children need to become fluent at retrieving answers to basic math facts.

As shown in Figure 1, at age 7, regular and special education students are nearly identical in the number of math facts they can recall from memory, however this changes by age 8 and this discrepancy increases as age increases. As students with math difficulty get older, they fall further and further behind their non-math-difficulty peers in the ability to recall basic math facts from memory (Hasselbring, Goin, & Bransford, 1988). Further, this lack of fluency interferes with the development of higher-order mathematical thinking and problem-solving.

Figure 1. A comparison of the number of fluent addition facts by age for regular and special education students

Number of Fluent Facts



DEVELOPING FLUENCY IN MATH-DELAYED CHILDREN USING TECHNOLOGY

To counteract the problem described above, educators have turned to technology with varying degrees of success to help students achieve fluency in math facts. Although it seems intuitive that using technology in a drill-and-practice format helps students develop the declarative fact knowledge, evidence suggests that this is not the case. In an early study by Hasselbring, Goin, and Sherwood (1986), it was found that computerized drill and practice was ineffective in developing declarative fact knowledge in students with math difficulty. The identified problem was that typical drill-and-practice software was designed in such a way that students were practicing "procedural counting" strategies instead of developing the ability to recall facts from memory. As a matter of fact, even studies that report reduced response latencies

as a result of the use of computerized drill and practice could not demonstrate that facts were being retrieved from memory, only that procedural counting time was reduced (Christensen & Gerber, 1990; Pellegrino & Goldman, 1987).

As a result of this research, Hasselbring and Goin (2005) developed an intervention paradigm called *FASTT* (Fluency and Automaticity through Systematic Teaching with Technology) designed to assist students in the development of declarative fact knowledge. The FASTT approach has been used successfully to develop mathematical fluency. It appears that the key to making the retrieval of basic math facts fluent is to first establish a mental link between the facts and their answers which must be stored in long term memory. FASTT embodies several unique design features to help develop these relationships. These 7 features include the following:

- 1. Identification of fluent and non-fluent facts;
- 2. Restricted presentation of non-fluent information;
- 3. Student generation of problem/answer pairs;
- 4. Use of "challenge times;"
- 5. Spaced-presentation of non-fluent information;
- 6. The appropriate use of drill-and-practice; and
- 7. Computer monitoring of student performance.

Each of the above features has been incorporated into a software program, called *FASTT Math* (2005) designed specifically to develop declarative fact knowledge.

Effectiveness of the FASTT Model

The principles embodied in FASTT Math were validated over several years of research with more than 400 students. This research with students with math difficulty has shown that the FASTT approach can be extremely powerful for developing fluency in the basic math facts. Generally, the findings show that when used daily, for about 10 minutes, most students with math difficulty can develop fluency in a single operation after approximately 100 sessions. The key to success appears to lie in the consistent use of the program. As expected, students who use the program regularly do much better than students who are irregular users.

As shown in Figure 2, the effects of using FASTT Math can be quite striking. In the first controlled study examining the use of the FASTT model, three groups of students were matched

for age, sex, and race. Two of the groups consisted of students with math difficulty and the remaining group consisted of students without math difficulty. In the experiment, one of the math difficulty groups (Math-Disabled Experimental), received an average of 54 ten-minute sessions on the FASTT software for addition, the other two groups (Non Math-Disabled and Math-Disabled Contrast) received only traditional fluency instruction delivered by their classroom teachers. As the data show, the students with math difficulty receiving instruction with the FASTT approach gained, on the average, 24 new fluent facts while their math-difficulty peers receiving traditional instruction gained no new facts and their non-math-difficulty peers gained only 8 new facts. Perhaps more impressive are the maintenance data. When the experimental students were tested 4 months after the posttest following summer vacation, the students regressed by only 4 facts, indicating that once facts become fluent, they are retained at a high level.

The results of this experiment have been replicated multiple times across all four operations. In all cases, when used consistently, the FASTT Math approach has a very positive effect on developing mathematical fluency in both students with and without math difficulty. Although FASTT Math is effective for all students needing assistance with developing fact fluency, it appears to be especially effective for students labeled as at risk and as learning disabled.

The result of this work demonstrates that students with math difficulty can be successful in attaining high levels of fluency in basic mathematical operations with the appropriate assistance of technology; however, this assistance must go beyond simple drill and practice if students have not stored the problem and the associated answer in long-term memory. FASTT Math was designed to help students create this network of problems and answers and then strengthen these relationships and increase fluency. For students who have already developed this stored network of relationships and do not rely on strategies such as counting to achieve a correct answer, then most any drill-and-practice activity will strengthen these relationships. There are many technology-based products available that will achieve this goal (see practice programs listed in the CITEd Mathematics Matrix under Building Computational Fluency).

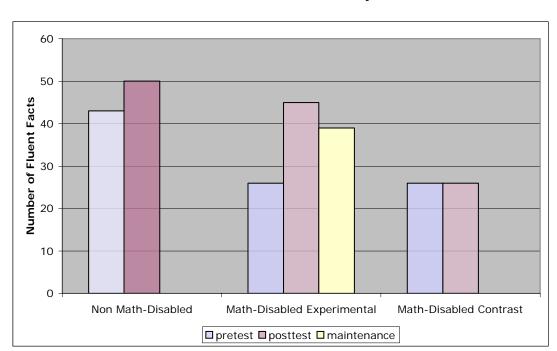


Figure 2. A comparison of the number of fluent addition facts for students with and without math difficulty

In sum, educators must know if students have developed a stored network of relationships containing basic problems and their answers or whether these relationships must be developed and stored in long term memory. This understanding is critical for educators to make an appropriate decision on which technology support will be most successful for students with math difficulty.

Converting Symbols, Notations, and Text

Scaffolding has been defined as a "process that enables a child or novice to solve a problem, carry out a task or achieve a goal which would be beyond his unassisted efforts" (Wood, Bruner, & Ross, 1976, p. 90). This support structure, or scaffold, is "faded," or reduced, over time as the student becomes more independently proficient. Scaffolding is rooted in the learning theories espoused by Vygotsky and Piaget (Greenfield, 1984; Rogoff & Gardner, 1984; Stone, 1993). The term "scaffolding" was first introduced by Wood, Bruner, and Ross (1976) in their article entitled, "The Role of Tutoring in Problem Solving" and the practice of scaffolding is primarily couched in the constructivism school of learning. Although scaffolding draws from the work of both Vygotsky and Piaget, it was influenced heavily by Vygotsky's (1935/1978) learning construct called the zone of proximal development (ZPD). This construct asserts that a

more knowledgeable person could help learners perform beyond their actual developmental level.

The original concept of scaffolding applied to the context of tutoring or one-on-one support. However, a classroom has multiple children with multiple ZPDs (Brown et al., 1993):

A single teacher is often providing scaffolding for up to 35 students at the same time, usually basing their help not on what any individual requires at the moment, but rather on what they believe most of the class needs in order to be successful (Puntambekar & Kolodner, 2005, p. 189).

As a result, students who face learning challenges often struggle to achieve parity with their peers, and students who grasp the concepts quickly are often bored and unchallenged by the review of material.

In order to help overcome the challenge of having multiple ZPDs in a classroom, computer tools have been developed to provide scaffolding to students on an individual basis. Computer tools can provide a form of scaffolding as the tools help offload some of the learner's cognitive task to the computer (Salomon, 1993). The goal of these tools is to enable the learner to eventually perform the task independently without the use of the tool (Salomon, 1993). As the students use these tools, they should begin to internalize this guidance, making the tools unnecessary.

One kind of cognitive task that can be offloaded to a computer is converting text, symbols, and mathematical notations. These tools can support students who have difficulty decoding text and symbols. By providing this individualized support, these tools are designed to take some of the burden off the teacher (who cannot work individually with 35 students simultaneously).

The problem with providing scaffolding to students in just the area of decoding is that it is not sufficient to address all students' needs. Researchers have found that one tool may not be enough to support a wide range of learners with multiple ZPDs and recommend that multiple tools be used in the classroom (Puntambekar & Kolodner, 2005). Thus, scaffolding tools can be created to support various learning needs, including the five other areas of technology use found in the CITEd Mathematics Matrix. For example, common to many computer-assisted instruction interventions for students with math difficulty are representation techniques (e.g. pictorial

instruction) to create mathematical representations and cognitive strategies (e.g. heuristic procedures) for problem solving (Xin & Jitendra, 1999).

Unfortunately, there are currently only a limited number of software packages that have been developed specifically to help students with difficulties decoding (see CITEd Mathematics Matrix for examples) as well as other learning needs and even fewer high-quality research studies identifying those that are effective. This lack of software and supporting research can be seen as an opportunity for researchers and math educators because there is currently a large quantity of non-special education-specific software that could be used to good effect for students with math difficulty. Using this new crop of software would provide researchers with new material for study, educators with the opportunity to expose their students to new tools, and students with the opportunity to gain exposure to new software that may be used to further maximize their potential.

Building Conceptual Knowledge and Understanding

Many students do not make direct conceptual links between concrete and tangible mathematical concepts, do not grasp representations of those concepts or relationships, and struggle to make the link from representation to abstractions. An important prerequisite for making these connections is that the declarative and procedural knowledge be taught within the same context(s) where it will be utilized in the future (i.e. the real world). This teaching enables that knowledge to be activated when needed (Bransford, Sherwood, Vye, & Rieser, 1986). Too often, special education students study mathematics by first learning isolated skills. Then they apply these skills by solving narrowly defined math problems that are purported to provide practice for these skills. Unfortunately, this strategy often leads to the practice of rote procedural skills and knowledge without students having a conceptual understanding of why the procedure is being used (i.e. restricted context).

Knowledge that is accessed only in a restricted set of contexts, even though it is applicable to a wide variety of domains, is known as *inert knowledge*. In order for it to be useful, knowledge cannot be inert. Students must understand how to conceptually apply their knowledge and procedures to real-world contexts. This type of learning should result in mathematical knowledge that is organized to *trigger* the conditions when that knowledge will be needed. Lesh (1981) proposed that the ability to use a new idea depends on the way it is connected to our prior

ideas and processes. Teaching a child a concept does not guarantee that it will integrate with other ideas that are already understood. As a result, situations in which the idea is relevant may not be recognized. The ability to retrieve useful information from memory appears to be especially challenging for children with learning disabilities or those who are at risk of school failure (Hasselbring et al., 1991).

One approach has been the use of video technology to create scenarios of real-world math problems (Cognition and Technology Group at Vanderbilt [CTGV], 1997). This approach to math instruction is called *anchored instruction* and has been used successfully with regular and special education students. This approach emphasizes the importance of anchoring or situating mathematical knowledge in meaningful, real-world applications. Video is used as the instructional medium because of its engaging characteristics. It can bring math to life for the students. This approach is not unique because it is delivered through video, but because it provides students with an opportunity to use the declarative and procedural knowledge gained in school to develop a conceptual understanding through the real-world application of this knowledge. This format helps students overcome their challenges in perceiving instances in which knowledge they already possess is useful. Since these environments are used within the context of teaching mathematical problem solving, they will be discussed in detail in the section on problem solving and reasoning.

Making Calculations and Creating Mathematical Representations

In this climate of high-tech software solutions for education, simple, older technologies that provide users with electronic means to make calculations, simplify and solve mathematical expressions and algebraic equations, often adaptive calculators, allow the user to focus on the conceptual and problem solving aspects of math. Technologies of this type often adaptive calculators that "serve as an equalizer in mathematics education" and "help students to more quickly and readily develop number sense, gain mathematical insight, and reasoning skills" (Pomerantz, 1997, p.2).

Although calculators (graphing and/or scientific) are relatively cost-effective tools and have been widely available for many years, educators have been slow to include their use on a daily basis in part due to misconceptions regarding their use in the educational curricula.

Pomerantz (1997) identified five common myths regarding the use of calculators by students in

the classroom. Among these myths are the notions that (a) calculators will promote student laziness, (b) students will not be stimulated/challenged if they use calculators, (c) using calculators impedes the development of basic mathematical skills, and (d) the use of calculators will create a dependency on technology. The common theme of these myths is that calculators will somehow hinder learning when, in fact, research has shown just the opposite to be true.

Research by Campbell and Stewart (1993) demonstrated that the use of calculators stimulated students to become problem solvers and strengthened their basic understanding of mathematical operations. Suydam and Brosnan (1993) reported that "research from over 100 studies indicated that the use of calculators (a) promoted achievement, (b) improved problem-solving skills, and (c) increased understanding of mathematical ideas. Suydam and Brosnan (1993) also reported that students who use calculators as part of a mathematics curriculum showed higher rates of information retention. Hembree and Dessart (1986) reported that students who used calculators demonstrated higher levels of math self-concept and, in general, exhibited a better attitude towards mathematics.

As effective as they may be, calculators are not the only portable devices that can be used with positive effect in the classroom. Handheld computers such as Palm devices and Pocket PCs offer an advantage of flexibility over traditional calculators. These handheld devices offer a great deal of computing power in a small package and a wide variety of software applications that can be used in a mathematics curriculum—database, spreadsheet, scientific probes/sensors, etc.

Overall, the use of calculators and handhelds in the classroom for both students with and without math difficulty presents an opportunity for educators to tap into a relatively cost-effective solution that has the potential to reap huge benefits in terms of student performance. As scientific calculators become more sophisticated and additional mathematics software becomes available for handheld devices, the power of individual student computing will increase dramatically. Given this, classroom curriculums must be designed dynamically allowing swift adaptation in order to take full advantage of the higher-order mathematical reasoning that will be within the grasp of future students.

Organizing Ideas

Research has shown that individuals who are skilled in math problem solving have something in common; they build a mental representation of the problem they are working

(Nathan, Kintsch, & Young, 1992; Pape, 2004). Additionally, skilled math problem solvers tend to classify or group problems by type and then look for known strategies that may be applied to that class of problem. The common elements of a problem that allow classification are known as the "underlying problem model." Research has found that the more effective students are at identifying the underlying problem model, the more successful they are at problem solving (Hegarty, Mayer, & Monk, 1995).

What this research effectively means is that individuals who are more effective at organizing their ideas about a math problem, in terms of identifying key features and elements, are ultimately more successful at solving problems. A graphic organizer is a visual representation of information. Past research has shown the use of graphic organizers to be an effective tool for math students (Jitendra, 2002; Willis & Fuson, 1988). Although the use of graphic organizers is widespread in education and the world of business (e.g. Microsoft PowerPoint presentation), math software that allows students to organize problems and help them identify underlying meaning has been limited; however, a new piece of software called GO Solve Word Problems has been created to help students organize math problems and discover their underlying structure. The software's interface allows students to organize the component parts of a math problem and then helps student to identify the relationships between the values and component parts of the problem.

TinkerPlots, developed with funding from a National Science Foundation grant is another piece of inquiry-based software that allows student-driven data organization and analysis (Steinke, 2005). Using TinkerPlots, students can graphically organize and construct data graphs by "stacking" iconic representations of numerical data. Although students can use their own data, TinkerPlots has several integrated rich datasets that may also be used.

As stated in the CITEd Mathematics Matrix, technologies that [allow the organization of ideas] provide a digital workspace for users to explore the connections among the text of problems to the concrete, representational and abstract concepts and apply these relationship to a wide range of problem solving strategies in real-world and mathematical situations. Given this Mathematics Matrix purpose of "Organizing Ideas," which includes the use of student-collected data in problem solving, software such as TinkerPlots and GO Solve Word Problems will undoubtedly open new doors to students' understanding of data management.

Building Problem Solving and Reasoning

Students with math difficulty find mathematical problem solving, particularly word problems, challenging for a variety of reasons as discussed by Babbitt & Miller (1996) in their review of literature. These challenges included misreading the problem, having difficulty detecting relevant versus irrelevant information, misidentifying the appropriate mathematical operation, making calculation errors, missing steps needed to carry out the problem, and having trouble organizing the information in the problem (Babbit & Miller, 1996). These challenges can be classified as problems with declarative, procedural, and conceptual knowledge. Students need all three types of knowledge to be able to solve problems. Problem solving requires students to know their basic mathematical facts, to execute the strategies and procedures needed to solve the problem, and to understand conceptually how to apply those facts and procedures. Without this conceptual understanding, there is no guarantee that the students will be able to apply this knowledge in meaningful ways when confronted with problem situations. Hasselbring et al. (1991) demonstrated that students often do not use prior knowledge spontaneously to solve problems unless they are explicitly informed about the relationship between that knowledge and the problem. In order for knowledge to be useful, students must understand how procedures can function as tools for solving relevant problems.

For example, when solving mathematical problems, students may have the mathematical knowledge and procedures they need but may be unable to use them because they lack the conceptual understanding that allows them to match their knowledge to the problem situation. The difficulty arises because their mathematical knowledge is either isolated chunks of knowledge or they are linked to conceptual understanding or models unconnected with the mathematics in the current problem. Without making these connections, students may be unable to detect when this knowledge applies to situations or when a strategy should be used during problem solving. Students who are unable to recognize situations in which their knowledge can be applied will likely be poor problem solvers. Knowledge that remains unused by learners even when it is relevant across several problem situations is wasted knowledge. For knowledge to be useful, it must be activated at the appropriate time. To enable students to become successful problem solvers, they must develop a working and dynamic relationship between declarative, procedural, and conceptual knowledge.

There are several approaches for helping students with mathematical problem solving. Some of these approaches target the declarative and procedural knowledge problems, some focus on students' difficulty with conceptual understanding, and others concentrate on improving students' reasoning and critical thinking. These approaches vary depending on the nature and complexity of the problem. For basic problems, some students are taught to search for key words. However, this approach has often been criticized because it does not always prompt the correct mathematical operation and creates a high likelihood of errors. Moreover, it does not require a real understanding of the problem situation and relegates the activity to an imitation of drill-and-practice (Porter, 1989). Another well-researched approach is to teach students with math difficulty cognitive strategies for solving problems. For example, Montague, Applegate, and Marguard (1993) studied the effectiveness of a cognitive strategy instruction. This 7-step strategy ranged from initial steps of learning to read the problem and develop hypotheses to the final steps of checking one's work. This method has proven to be effective in helping students with math difficulty. Similar types of instructional strategies have been applied to technologybased interventions. The majority of these interventions are still in the prototype phase and thus are not yet available commercially.

One prototype intervention that incorporated a word problem-solving strategy into a computer tutorial program was developed by Shiah, Mastropieri, Scruggs, and Fulk (1994–1995). The 7-step strategy included (1) reading the problem, (2) thinking about the problem, (3) deciding the operation, (4) writing the numerical sentence, (5) calculating the math operation, (6) labeling the answer, and (7) checking work. These steps were incorporated into the program as text pop-up buttons that would prompt the student to complete each step. The researchers compared different versions of this computer program with 30 elementary school students classified as having learning disabilities. They found that students who used the computer program with the 7-step strategy significantly improved their word problem solving when tested online, but did not make significant improvements on the traditional paper-and-pencil test of word problems. Moreover, they did not find a significant difference in improvement between students who used the strategy and the control group. They hypothesized that this may have been caused by the fact that the control group had more time to solve problems because the strategy group required more time to learn the strategy. Future research may want to re-evaluate this type

of computer program in a setting where all groups have equivalent time to solve the word problems.

Another prototype intervention designed to assist students with solving basic word problems was a program that focused on teaching students different methods of problem representation. Stellingwerf and Van Lieshout (1999) compared four different versions of a computer program with 140 students (between 9 and 13 years old) from schools for children with learning problems and mild mental retardation. One version prompted the students to represent the problem as a numerical expression. A second version asked students to represent the problem visually by selecting icons. A third version combined both techniques, and a fourth version just presented students with problems without either representation strategy. There was also a control group that did not use any software. Although all children improved in word problem solving from the computer instruction, the versions of the computer program that prompted students to write number sentences were most effective. However, this effect was only found for students who were relatively more competent in solving word problems. The students in the study had previous experience solving word problems, and thus, it may be that iconic representations are more helpful for students who are less familiar with solving word problems.

An entirely different approach that has proved effective in helping students with solving more complex, real-world problems is the aforementioned *anchored instruction* (CTGV, 1992). These anchored instruction environments combine video and audio technologies in a story format. Because students identify with the characters in the story, they are situated in the problem and motivated to find a solution. It is important to note that not all students may relate to the same story, or anchor, so it is important for educators using this technique to tailor anchoring content to meet the needs of their specific group of students.

The numerical data needed to solve the problem is subtly presented during the story, and the students must uncover the pertinent information needed to solve the problem. Instead of modeling a solution, the generative format encourages students to discover the final outcome and prompts them to be active participants in the learning process. These anchored instruction environments typically include two videos that focus on related content areas. For example, these videos might focus on the topics of measurement, fractions, and money within the context of a problem scenario. Moreover, they are interdisciplinary and connect math to other subject areas, such as science and social studies. Overall, these video-based adventures provide a motivating

and realistic context for problem posing, problem solving, and reasoning within specific math topics for students having learning problems.

Early research and development efforts on anchored instruction began at the Cognition and Technology Group at Vanderbilt (1992) with the *Woodbury Jasper Series*. Bottge and Hasselbring (1993) were the first to develop these video-based adventures specifically for students with learning disabilities. More recently, Bottge and his colleagues at the University of Wisconsin have explored a new approach called enhanced anchored instruction. This approach combines video-based anchors with applied problems, which allow students to physically build and test out their solutions in technology education classes (e.g. Bottge, 1999). Within enhanced anchored instruction, the videos are used as part of a broader curriculum that incorporates other forms of math instruction, such as direct instruction or cognitive strategy instruction, along with hands-on building activities.

One example of an anchored instruction environment is a video entitled "Fraction of the Cost." This video was developed by the Wisconsin Center for Education Research. It begins with characters Cindy, Ryan, and Michael walking by a skateboard shop. They go into the store to look at skateboards, and they discover an indoor skateboarding park in the back of the store. While running around in the park, Michael has an idea about getting his own skateboard ramp. They inquire with the storeowner about buying one, but the storeowner says that the ramps are not for sale. Instead, he provides them with a link to a Web site that provides a plan for building a ramp. After examining the plans, the friends decide that the ramp won't be too difficult to build because the plans are similar to the compost bin they built in their technology education class. The question is whether they have enough money to buy the materials to build the ramp. This video and accompanying lesson plans can be retrieved from the Wisconsin Center for Education Research Web site, http://www.wcer.wisc.edu/TEAM/index.html.

In order to solve the over-arching problem, students must first gather the relevant facts. All factual information is found in the video. These facts appear in the video in several different ways, all of which parallel natural settings. For example, the cost of lumber is provided in an advertisement in the Sunday paper. The viewer learns about the amount of money the kids have to spend on the materials through the kids talking about their savings. The skateboard dimensions are detailed in a schematic plan from a Web site. The solution requires that the students be able to use their declarative and procedural knowledge in several areas, including

money, measurement, whole numbers, and fractions. More importantly, they must understand where and how procedural knowledge in each of these areas is useful and how it is applied. In enhanced anchored instruction, students are then asked to apply their procedural knowledge gained while solving the video problem to a real-life problem of building a bench or a hovercraft in their technology education class.

The researchers have examined the effectiveness of anchored instruction environments, like the "Fraction of the Cost" video, on the mathematical problem solving of students with learning difficulties. These environments were found to be more effective than other problem-solving instruction in helping students solve complex, contextualized video problems (Bottge, 1999; Bottge & Hasselbring, 1993; Bottge et al., 2002; Bottge et al., 2004). Students who used the anchored instruction environment also performed significantly better than groups that received other problem-solving instruction on a variety of transfer tasks, including complex text problems (Bottge, 1999; Bottge et al., 2002), contextualized video problems (Bottge & Hasselbring, 1993), and applied construction problems (Bottge et al., 2004). Only one study did not find this type of environment to have a significant effect on transfer for complex text problems (Bottge & Hasselbring, 1993).

Overall, students in the anchored instruction group were able to transfer skills learned during instruction to a variety of problems. These findings indicate that a much more robust relationship between these students' declarative, procedural, and conceptual knowledge was developed. On the other hand, the majority of these studies found that anchored instruction environments were no more effective than other problem-solving instruction on improving students' ability to solve traditional word problems (Bottge, 1999; Bottge & Hasselbring, 1993; Bottge et al., 2003; Bottge et al., 2001).

In general, research has found that anchored instruction environments are very effective for students with learning disabilities within remedial education settings (Bottge, 1999; Bottge & Hasselbring, 1993; Bottge et al., 2003; Bottge et al., 2001). On the other hand, when students with learning disabilities were taught within inclusive classroom settings, anchored instruction environments did not improve their scores to the same extent as when students were separated into remedial and general education classes (Bottge et al., 2002; Bottge et al., 2004). Bottge et al. (2004) recommend that, in inclusive environments, anchored instruction environments be used in conjunction with individualized or pull-out instruction.

In a different type of study, Fuchs, Fuchs, Hamlett, and Appleton (2002) compared computer programs that incorporate contextualized videos to small-group tutoring that provided instruction on problem-solving rules and transfer to students with math difficulty. Students in the tutoring group were taught to look for similarities between novel and familiar problems. On real-world problem-solving and transfer tasks, both groups improved to a similar extent. For the transfer tasks, both groups performed significantly better than the control group. However, for simpler word problems, students in the tutoring groups did significantly better than the computer group. These results were replicated in a later article by Fuchs and Fuchs (2005). These findings, along with the results from Bottge and his colleagues, indicate that anchored instruction or contextualized environments may be more beneficial for teaching students how to solve more complex, real-world mathematical problems, whereas other approaches may be more suitable for more traditional word problems.

As shown from this review of technological innovations for improving the mathematical problem solving of students with disabilities, there is a great need for empirical studies of software programs that are commercially available for use in inclusive and special education classrooms. It is also critically important for researchers to collaborate with software companies to further develop their research prototypes, so they can be made available to a wider audience. Although the majority of the technology-based interventions reviewed are still in the prototype phase, the findings may be helpful for educators in looking for software with specific features that have proven to be effective in helping students with mathematical problem solving.

SUMMARY

In sum, the differential in mathematics performance between students with and without math difficulty that has been observed over many years remains, yet the commitment to improving outcomes for students with math difficulty continues to grow. One strategy that needs additional attention involves the use of technology designed to teach mathematical concepts in non-traditional ways. At present, the sheer quantity of educational software and other tools that are available for teachers to use in the classroom is significant. Additionally, the cost of much of this hardware and software is relatively low. Nevertheless, while the commitment to improving the math performance of students with math difficulty is strong and the technology to help educators accomplish this goal is readily available, there is a paucity of research related to the

effectiveness of these approaches. Further, there is a dearth of research related to the identification of best practices necessary to effectively implement math instruction with the help of technology.

One major goal of educators of students with math difficulty should be to conduct ongoing research to determine the best use of existing technology for enhancing mathematical learning. Further, educators and researchers should work closely with developers and publishers of new hardware and software and conduct high-quality research targeted at identifying effective practices that accompany the use of new products. In this paper we have attempted to identify important areas in need of research and development and to examine a variety of technologies that can enhance the mathematical learning of all students, but especially those students with math difficulty. Hopefully, we have identified areas of need that will serve as a guidepost for future research and development activities.

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