

Effective Mathematics Teaching from Teachers' Perspectives

National and Cross-National Studies

Jinfa Cai , Gabriele Kaiser, Bob Perry
and Ngai-Ying Wong (Eds.)



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TABLE OF CONTENTS

Preface	vii
1. What is Effective Teaching? A Study of Experienced Mathematics Teachers from Australia, the Mainland China, Hong Kong-China, and the United States <i>Jinfa Cai, Bob Perry, Ngai-Ying Wong, and Tao Wang</i>	1
2. What is Effective Mathematics Teaching? International Educators' Judgments of Mathematics Lessons from the TIMSS 1999 Video Study <i>Karen Bogard Givvin, Jennifer Jacobs, Hilary Hollingsworth, and James Hiebert</i>	37
3. What is "Good" Mathematics Instruction? Mathematics Teachers' Individual Criteria for Instructional Quality and Attributions for Instructional Success <i>Sebastian Kuntze and Franziska Rudolph-Albert</i>	71
4. Good Mathematics Teaching and the Role of Students' Mathematical Thinking: High School Teachers' Perspectives <i>Patricia S. Wilson and Kanita K. Ducloux</i>	93
5. In Search of Effective Mathematics Teaching Practice: The Malaysian Mathematics Teachers' Dilemma <i>Chap Sam Lim</i>	123
6. What are Teacher's Beliefs about Effective Mathematics Teaching? A Qualitative Study of Secondary School Teachers in Germany <i>Katja Maass</i>	141
7. Examining the Nature of Effective Teaching through Master Teachers' Lesson Evaluation in China <i>Rongjin Huang and Yeping Li</i>	163
8. Teachers' Beliefs, Instructional Practices, and Culture: Understanding Effective Mathematics Teaching in the Philippines <i>Catherine P. Vistro-Yu and Rosemarievic Villena-Diaz</i>	183
9. Effective Mathematics Teaching in Finland Through the Eyes of Elementary Student Teachers <i>Raimo Kaasila and Erkki Pehkonen</i>	203

TABLE OF CONTENTS

10. On the Quality of Mathematics Lesson: Do Elementary Mathematics Teachers Have Similar Views as Students and Their School? <i>Yeping Li, Gerald Kulm, Rongjin Huang, and Meixia Ding</i>	217
11. Beliefs about Mathematics and Effective Teaching among Elementary Mathematics Teachers in Hong Kong <i>Qian-Ting Wong, Ngai-Ying Wong, Chi-Chung Lam, and Qiao-Ping Zhang</i>	235
12. “Effective Teaching of Mathematics” by Teachers, for Teachers: An Australian Case Study <i>Will Morony</i>	259
13. The Presidential Award for Excellence in Mathematics Teaching: Setting the Standard <i>Iris R. Weiss, P. Sean Smith, and Sharon K. O’Kelley</i>	281
14. Studying Effective Teaching from Teachers’ Perspectives: The Journey has Just Begun <i>Jinfa Cai, Tao Wang, Ning Wang, and Tiffany Garber</i>	305
Editors and Contributors	319

PREFACE

INTRODUCTION

Classroom instruction is accepted as a central component for understanding the dynamic processes and the organization of students' mathematical thinking and learning (Cai, 2004; Gardner, 1991; Rogoff & Chavajay, 1995). Because classroom instruction plays such a central role in students' learning, researchers have long tried to characterize the nature of the classroom instruction that maximizes students' learning opportunities (Brophy & Good, 1996; Floden, 2001). Teachers are central to classroom instruction in mathematics and have a major impact on students' learning. Consequently, if our aim is to improve students' learning of mathematics, one fruitful line of endeavor is to investigate the characteristics of effective mathematics teaching.

Much of the early research on the effectiveness of mathematics teaching focused on teacher knowledge of mathematics (Thompson, 2004). Teachers' beliefs about mathematics, mathematics learning and mathematics instruction can also impact on teachers' instructional practices (Beswick, 2007; Leder, Pehkonen, & Törner, 2002; Wilkins, 2008), although the contextual nature of beliefs means that it is unwise to expect consistent links between beliefs and practice. While teachers' beliefs have been described by Pajares (1992, p. 307) as "a messy construct", their influence on instruction is sufficiently accepted to warrant further investigation.

This book focuses particularly on how teachers view effective teaching of mathematics. One of the unique features of the book is this reliance on the views of teachers as the primary sources of data for each chapter. Hence, teachers' voices are heard and celebrated throughout the chapters. Another feature of the book is the geographical and cultural spread of the teachers (and authors) involved in the development of the chapters. While this is a reflection of the spheres of contact in which the editors have worked over many years, it also reflects a particular strategy through which the editors aimed deliberately to include the views of teachers from different cultural backgrounds, taking into account that beliefs on effective mathematics teaching and its features are highly culturally dependent.

ORIGIN OF THE BOOK

The initial impetus for this book comes from the work of its editors over the last 15 years in the areas of effective mathematics teaching; teacher beliefs about mathematics, mathematics learning and mathematics teaching; and cross-cultural comparisons of mathematics teaching and learning. Both alone and in conjunction with each other, we have compiled an extensive collection of publications in these areas.

In October, 2002, all of us were invited participants in the 13th ICMI Study Conference held at the University of Hong Kong. This study, *Mathematics Education in Different Cultural Traditions: A Comparative Study of East Asia and*

the West (Leung, Graf, & Lopez-Real, 2006), brought together many mathematics education researchers to consider how cultural traditions might impact on mathematics teaching and learning. All of the editors of this book were invited to present papers at the Study Conference (Cai, 2006; Kaiser, Hino, & Knipping, 2006; Perry, Wong, & Howard, 2006; Wong, 2006) and during discussions the seeds for this book were sown.

Work continued on the impact of cultural traditions on mathematics teaching with three of the editors (Cai, Perry, and Wong) conducting a study during 2004/2005 which gathered data from outstanding elementary/middle school teachers in Australia, Mainland China, Hong Kong SAR-China and the United States of America on what these teachers perceived an effective teacher of mathematics to be and to do. This study was presented at the 2006 conference of the International Group for the Psychology of Mathematics Education (PME) in a working group entitled *What is effective mathematics teaching? East meets West*.

The international study of what teachers perceived as effective mathematics teaching was published as an issue of *ZDM – The International Journal on Mathematics Education* (Volume 39(4), 2007) entitled *What is Effective Mathematics Teaching? A Dialogue between East and West*. All the editors of this book contributed to this special issue.

Growing from both the PME Working Group and the *ZDM* issue, invitations were extended to colleagues working in the area of teachers' views of effective mathematics teaching and this book is the result. It is a collection of chapters themed to teachers providing their views about what they see as effective mathematics teaching. There have been many fine studies of cultural differences among mathematics teachers and teaching, most of them based on the observation and detailed analysis of classrooms and lessons (for example, Clarke, Keitel, & Shimizu, 2006; Stigler & Hiebert, 1999). This book, however, represents the first purposeful cross-cultural collection of studies concerning teachers' views of effective mathematics teaching.

Research is presented from many parts of the world representing both Eastern and Western cultural traditions. Included are Australia, China – both Mainland and Hong Kong SAR, Czech Republic, Finland, Germany, Japan, Switzerland, Malaysia, Netherlands, Philippines, and United States of America. National or cross-national studies are presented in 12 chapters and are compared in a final chapter that discusses the methodologies and findings from each of these studies. Always, the emphasis is on what practicing teachers see effective mathematics teaching to be.

Another chapter which does not report a research study is also included in the book. This chapter reports on an initiative taken by the Australian Association of Mathematics Teachers to develop standards by which effective mathematics teaching might be identified and celebrated. In many ways, this chapter provides a culmination for the research studies reported in the book, showing just how teachers' views of effective mathematics teaching can be used not only to improve teaching and learning but also to improve the image of teaching within societies.

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The editors would like to express their thanks to many people for assisting them in bringing this book to fruition. First and foremost, thanks are due the many chapter authors who have diligently worked at preparing, revising and finalizing their chapters within timelines that were, probably, unreasonable. Secondly, thanks to the professionals at Sense Publishers, particularly Peter de Liefde, for their patience and expertise in leading us through the production of the book. Thirdly, thanks to our families who have had to endure our absences while we undertook the task of putting this book together. Finally, we should thank each other. It is not easy to bring a book together when the four authors are on four different continents in the world. However, it is very gratifying to have worked in a team so dedicated to the task, in spite of the local distractions that are always present in busy lives. We hope that the book will provide future readers and scholars with the stimulus to take the ideas presented and expand on them in ways that help improve mathematics education for children, teachers and researchers in both the East and the West.

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PREFACE

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JINFA CAI, BOB PERRY, NGAI-YING WONG, AND TAO WANG

WHAT IS EFFECTIVE TEACHING?

*A Study of Experienced Mathematics Teachers from Australia,
the Mainland China, Hong Kong-China, and the United States*

INTRODUCTION

While family and students' out-of-school experiences play important roles in their learning (Cai, 2003; Lave, 1988), students acquire much of their knowledge and develop their thinking skills from classroom instruction. Thus, researchers have long tried to understand the nature of classroom instruction to maximize students' learning opportunities. Because classroom instruction is a complex enterprise (Leinhardt, 1993), researchers have attempted to identify its important aspects in order to investigate the kinds of classroom instruction that are effective in fostering students' learning (Brophy & Good, 1996; Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Good, Grouws, & Ebmeier, 1983; Hiebert & Wearne, 1993; Perry, VanderStoep, & Yu, 1993). One of the important aspects of classroom instruction that has been considered for such investigation is on the beliefs¹ of teachers about mathematics, mathematics learning and mathematics teaching (Battista, 1994; Beswick, 2007; Leder, Pehkonen, & Törner, 2002; Pehkonen & Törner, 1998; Perry, Howard, & Tracey, 1999; Thompson, 2004; Wong, Marton, Wong, & Lam, 2002).

However, beliefs about mathematics and its learning and teaching are not the only teacher beliefs that need to be considered when we are looking for influences on the effectiveness of teaching (Gates, 2006; Sztajn, 2003). Mathematics educators recently have begun to examine other sets of beliefs that influence mathematics teaching practices. Skott (2001) showed how beliefs not directly related to mathematics teaching also help one understand mathematics teachers' practices. In his study, he considered micro-aspects of the social contexts of mathematics classrooms. He presented the teacher's overarching concern about students' self-esteem as justification for mathematics teaching episodes.

It seems that teachers' beliefs about their students and how the students are situated in social contexts that may not be well understood by the teachers are closely related to the students' motivation to learn, and their performance in mathematics (Philippou & Christou, 2002; Zevenbergen, 2003). This, in turn, seems to be related to the effectiveness of the mathematical instruction provided (McLeod, 1992; Pehkonen & Törner, 1998).

The idea that teachers' instructional practices are influenced by both their cultural beliefs and their conceptions of effective teaching is not new (Cai, 2005; Perry, Wong, & Howard, 2006; Stigler & Hiebert, 1999). In fact, teachers do draw upon their cultural beliefs as a normative framework of values and goals to guide their teaching (Rogoff, 2003). A teacher's manner of presenting mathematics is an indication of what he/she believes to be most essential, thereby influencing the ways that students understand and learn mathematics (Cai, 2004; Cooney, Shealy, & Arvold, 1998; Greeno, 1987; Thompson, 1992). Some researchers are even calling for changes in teachers' cultural beliefs concerning who should learn mathematics, what mathematics should be learned, and how mathematics should be taught, in order to change mathematics teaching practice in classrooms (Tirosh & Graeber, 2003).

Over the last two decades, researchers have developed theories about teachers' beliefs and the way these beliefs impact teachers' classroom practice (see, for example, Leder et al., 2002; Thompson, 1992). However, these theories were mainly developed from studying Western mathematics teachers in Western mathematics classrooms (Biggs, 1994; Graf, Leung, & Lopez-Real, 2006; Wong, 2004). Only in recent years have researchers started to cross-culturally explore teachers' beliefs and look at how teachers' culturally constructed beliefs impact their teaching and the learning of their students (Cai, 2004, 2005; Gao & Watkins, 2001; Stigler & Hiebert, 1999). However, in all of these works, there are few studies examining teachers' beliefs about effective teaching (an exception to this is the recent special issue, Volume 39, Number 4 in 2007, of *ZDM – The International Journal on Mathematics Education*) containing papers by the authors of this chapter.

The study reported in this chapter examines teachers' beliefs about effective teaching in mathematics from a cross-cultural perspective. It is designed to address the following fundamental question: What is effective teaching for teachers in Eastern and Western cultures? From a cross-national perspective, this study aims to develop a deeper understanding of teachers' cultural beliefs concerning effective mathematics instruction.

Theoretically, the findings from this study will make a significant contribution to our understanding of teaching and teachers' belief systems from a cross-cultural perspective. Practically, such an investigation should provide insightful information about what can be learned from instructional practices in different countries in order to improve students' learning of mathematics. This study not only helps us understand the relationship between teachers' conceptions and instructional practices from an international perspective, but it also helps us interpret the differences in the teaching and learning of mathematics found in previous cross-national studies (e.g., Cai, 2000).

Systems of Beliefs about Effective Mathematics Teaching

Ernest (1989) identified three main components of teachers' mathematical belief systems: teachers' views of the (1) nature of mathematics, (2) features of

mathematics teaching, and (3) process of learning mathematics. According to Ernest, the teachers' view of the nature of mathematics is the most fundamental because it impacts two other closely related beliefs about mathematics teaching and learning, although it has also been suggested that the components are inter-related in many ways (Perry et al., 1999; Speer, 2005; Thompson, 1992). While there are many ways in which teachers' beliefs about mathematics and its teaching and learning have been characterized (Kuh & Ball, 1986; Leder et al., 2002; Lerman, 1990), in this study, we have used Ernest's ideas of teachers' belief systems as our primary organizer for designing the study and conducting the data analysis.

View of the nature of mathematics. A teacher's conception of the nature of mathematics can be viewed as that teacher's conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences concerning the discipline of mathematics. Those beliefs, concepts, views, and preferences constitute the beginnings of a philosophy of mathematics, although, for some teachers, they may not be developed and articulated into a coherent philosophy. For many people, mathematics is a discipline characterized by abstract knowledge, accurate results, and strong logical procedures. However, people's views vary greatly about the origin of this abstract knowledge system and how people can partake of it.

According to Ernest (1989), teachers view mathematics from at least two perspectives: functional and structural. From a functional perspective, mathematics is seen as "a bag of tools ... made up of an accumulation of facts, rules and skills" (p. 250). Ernest characterized this view as "instrumentalist." Teachers who view mathematics from a structural perspective see it as a "unified body of knowledge, a crystalline realm of interconnecting structures and truths, bound together by filaments of logic and meaning" (p. 250). Ernest called the second view the Platonist view. While the instrumentalists pay more attention to the functions of mathematics knowledge on the external world, Platonists put their emphasis on the complicated internal structure of the knowledge itself.

In practice, an individual teacher's beliefs about the nature of mathematics often include changeable combinations of instrumentalist and Platonist views. The effects of these beliefs on teacher's actions in relation to teaching mathematics are impacted by the contexts in which the teachers are working. Using Green's (1971) concept of the relative centrality of a belief within a teacher's belief system, Beswick (2007) has claimed that

The relative centrality of an individual's beliefs will vary from context to context. Failure to enact a particular belief evident (via words and/or actions) in one context or another, can thus be seen as the result of different beliefs taking precedence in the different situations (p. 97).

View of the mathematics teaching and learning. Teachers' beliefs about teaching mathematics, according to Thompson (1992), can be revealed in following aspects: desirable goals of the mathematics program, a teacher's role in teaching, appropriate classroom actions and activities, desirable instructional approaches and

emphases, and legitimate mathematical procedures. Similarly, teachers' beliefs about the learning of mathematics cover the processes of learning mathematics, what behaviors and mental activities are involved on the part of the learner, and what constitute appropriate and prototypical learning activities (Thompson, 1992).

Kuhs and Ball (1986) identified four views concerning teachers' beliefs about mathematics teaching and learning. These have been summarized by Speer (2005) in the following way.

The "learner-focused" view centers on the learner's personal construction of mathematical knowledge through active involvement in doing mathematics. The teacher's role is as a facilitator of student learning. The second view, "content-focused with an emphasis on conceptual understanding," focuses on the logical relations among mathematical ideas. "Content-focused with an emphasis on performance" is similar to the previous one in its focus on mathematical content, but emphasizes rules and procedural mastery. The fourth view, "classroom-focused," emphasizes classroom activity that is structured, efficiently organized, where teachers present material clearly and students practice individually (p. 366).

Ernest (1989) has proposed three teaching models to reflect the various roles a teacher might play in a classroom: instructor model, explainer model, and facilitator model. The intended outcome for an instructor often focuses on student skill mastery and correct performance; for an explainer, conceptual understanding with unified knowledge; and for a facilitator teacher, student confidence in problem posing and solving. There are clear links among the three characterizations of beliefs about mathematics teaching and learning the two views held about mathematics itself. These commonalities will be used in the analysis of data from the current study.

Cross-cultural comparisons of teachers' beliefs. The formative work around the development of understandings of teachers' belief systems about mathematics, mathematics teaching and mathematics learning has been conducted in Western countries by Western researchers. Recently, there have been some attempts to understand teachers' beliefs from cross-cultural perspectives. A few studies have demonstrated the value and feasibility of cross-national studies in investigating teachers' beliefs about effective teaching (Cai, 2004, 2005; Marton, Tse, & dall'Alba, 1996; Stigler & Hiebert, 1999; Perry et al., 2006; Perry, Vistro-Yu, Howard, Wong, & Fong, 2002). For example, Marton et al. (1996) examined teachers' views of memorization and understanding and suggested that in Western countries memorization and rote learning are generally considered the same. Western educators also believe that memorization does not lead to understanding. However, through extensive interviews with 20 Chinese teacher educators, Marton et al. (1996) provided a new way of seeing the relationship between memorization and understanding. For Chinese educators, memorization does not necessarily lead to rote learning; instead, it can be used to deepen understanding.

Recent studies (Cai, 2004, 2005; Cai & Wang, 2006) examined Chinese and U.S. teachers' cultural values of representations in instruction. They found that Chinese and U.S. teachers hold different curricular expectations. Chinese teachers expect 6th graders to be able to use equations to solve problems, but U.S. teachers have that expectation only for 7th or 8th grade students. The fact that U.S. and Chinese teachers have differing curricular expectations is not surprising since the curricula of the two countries are very different. However, on a deeper level, the differences in expectations may reflect the differences in cultural beliefs about mathematics. Although both the Chinese and U.S. teachers agreed that mathematics has wide applications in the real world, the true beauty of mathematics for Chinese teachers was its purity, generality, and logic. Thus, a solution strategy that lacks generality (e.g., a visual approach) should be discouraged. In contrast, U.S. teachers heavily emphasized the pragmatic nature of mathematics, leading, at least for some of the U.S. teachers, to a belief that as long as it works, students can choose whatever representations and strategies they like.

Stigler and Hiebert (1999) found that, to a large extent, the different beliefs held by Eastern and Western teachers explain the different mathematics instructional patterns they observed in East Asian and U.S. classrooms. For example, they found that Asian teachers teach mathematics in a coherent way because they believe that mathematics is a set of relationships between concepts, facts, and procedures. In contrast, Stigler and Hiebert describe U.S. teachers' understanding of school mathematics as "a set of procedures" and "skills." Based on classroom observation, Stigler and Perry (1988) found that U.S. teachers tend to believe that young children need concrete experiences in order to understand mathematics. Chinese teachers, however, believe that even young children can understand abstraction and that concrete experience only serves as a mediator for understanding (Stigler & Perry, 1988).

These studies have indicated the value and feasibility of investigating teachers' beliefs about teaching mathematics from an international perspective. However, there is more to be done, particularly in terms of how teachers from the East and West view and practice effective mathematics teaching. The aim of the current study is to contribute substantially to this work.

METHODS

When teachers' views about effective teaching are studied, the first problem is to determine the criteria to be used for judging teaching effectiveness (Cai, 2005). In this study, we take the position that the quality of mathematics instruction can be judged by two criteria: desirable outcomes in students' learning and the processes that yield those desirable learning outcomes. This position has strong support from the extant literature on mathematics learning and research on teaching and teacher development (e.g., Bransford, Brown, & Cocking, 2000; Cobb, 1994; Floden, 2001; Hatano, 1993; Schoenfeld, 2000).

In this study, we are particularly interested in teachers' views about the characteristics of effective teachers, characteristics of effective lessons, memorization and understanding, and the role of practice in students' learning. The study adopts

a descriptive approach in order to understand teachers' belief systems in different cultures and does not judge or evaluate these beliefs. The study builds on previous work by the authors which has tried to understand cultural differences in teachers' beliefs on the nature of mathematics, the teaching and learning of mathematics in general, and effective mathematics teaching in particular (e.g., Cai, 2005; Cai & Wang, 2006; Perry et al., 2006).

It is clear that there is no single educational region that can represent the East (or the West), and we cannot draw a line on the globe to divide the East from the West. However, the purpose of this study was to make cultural contrasts about the beliefs of effective teachers of mathematics. Australia, the Mainland China, Hong Kong SAR², and the United States were selected for the study because they represent a spectrum of Eastern and Western cultures. The U.S. can be considered Western and the Mainland China the Eastern. Hong Kong is clearly influenced by Chinese culture, but it is also influenced by the more than a century of Western (British) colonization. Australia is also strongly influenced by British colonization but, over the last 60 years, has grown into one of the world's most multicultural nations and represents a highly diverse cultural melting pot. To some extent, Hong Kong and Australia can be considered to be some where between the Eastern and Western extremes exemplified by the Mainland China and the U.S., respectively, with Hong Kong SAR "closer" to China than Australia and Australia "closer" to the U.S. than Hong Kong SAR.

Effective teachers of mathematics were identified in each jurisdiction using local definitions of effectiveness. Each of the selected teachers was interviewed using semi-structured questions in order to understand each teacher's views about mathematics, teaching mathematics, and learning mathematics. Through semi-structured interviews, we can understand not only what teachers believe, but also why they hold these beliefs.

Selection of Teachers

Thirteen Australian teachers, 9 from China, 12 from Hong Kong, and 11 from the U.S. were selected for the study. The U.S. teachers were selected from Delaware, Milwaukee (WI), and Philadelphia (PA). Chinese teachers were selected from Guiyang, Guizhou, a developing province in the southeast of China, and from the Hong Kong SAR. The Australian teachers were selected from three states – Tasmania, New South Wales and Victoria – and the Australian Capital Territory. There is no suggestion that the sample of teachers in this study is representative of the effective teachers of mathematics in their regions³. Rather, in each case, effective teachers were identified through processes appropriate to the local context.

All the selected teachers were distinguished mathematics teachers in their regions. Local criteria were used to select the teachers. In the U.S. for example, teachers who had received recognition for their teaching excellence, such as the presidential award in teaching or the teacher of the year award were chosen. For teachers from Guizhou, China, those who had been recognized as special class or first class teachers (the highest ranks for the teaching profession in China), were selected.

As for Hong Kong SAR, though such a special class/first class system is not established, all teachers except one possessed over 10 years of teaching experience, and most of them were actively involved in professional bodies; as well, three had earned Masters of Education degrees. At the time of selection of the teachers, Australia did not have a process for recognizing excellence in mathematics teaching except through the state and national mathematics teaching professional associations. The Australian teachers for this study were selected after a request was made to the Australian Association of Mathematics Teachers (the national professional association) and the Mathematics Association of New South Wales for them to nominate, from their membership, elementary school teachers who were excellent teachers of mathematics. The association made the nominations and checked with the nominees to be sure that they were willing to be involved in the study. Tables 1-4 (in Appendix) show more detailed background information about the selected teachers in the study.

It goes without saying that teachers are recognized as distinguished teachers of mathematics in their regions because they represent the culturally accepted values of effective mathematics instruction. Therefore, the inclusion of distinguished mathematics teachers may facilitate the process of identifying cultural values and criteria of effective mathematics teaching.

Interview Questions and Data Analysis

Semi-structured interviews were conducted and three sets of interview questions were used in this study.

About Mathematics:

In your view, what is mathematics? What is the substance of it?

Some people believe: A lot of things in mathematics must simply be accepted as true and remembered and there really aren't any explanations for them. What do you think?

Some people believe: Mathematics is abstract; therefore, we need to help students think abstractly. What do you think?

About Learning:

Many people believe: Learning mathematics with understanding is essential. What do you think? What is "understanding" anyway? What do you think a teacher should do to help students learn mathematics with understanding?

Many people believe: In order to help students learn mathematics with understanding, concrete experiences are necessary. What do you think? What concrete experiences do they refer to?

What role does memorization play in students' learning of mathematics?

What role does practice play in students' learning of mathematics?

About Teaching:

We all know some teachers are more effective than others in teaching. In your view, what characteristics does an effective teacher have?

We also know some lessons are better than others. What is an ideal, excellent lesson? What characteristics should an ideal, excellent lesson have?

How should teachers use manipulatives and/or concrete representations in their teaching?

Interviews were either videotaped or audiotaped and transcribed. In data analysis, we adopted three phases to code and analyze transcribed data. Firstly, researchers began with open coding of all transcribed interview data. The purpose of this open coding phase was to find unanticipated salient examples of cultural beliefs from the teachers. Second, we re-examined all the data using a start list of codes that were developed to specifically address the research questions about teachers' beliefs on the nature of mathematics as well as the teaching and learning of mathematics. We looked for commonly expressed themes in teachers' responses. Finally, we compared the similarities and differences among teachers' beliefs from the four regions. This process helps us develop a grounded theory (Miles & Huberman, 1994; Strauss & Corbin, 1990) to understand the cultural differences about the teachers' beliefs.

Translation Equivalence

In a cross-national study involving interviews in two languages, it is absolutely essential to ensure the equivalence of the two language versions of the instruments. To address this, a process of English-back translation was used. In this process, two people, each literate in both Chinese and English, contributed to the translation of the instruments. The interview questions were originally in English. The first person translated them from English into Chinese. The second person then translated the Chinese back into English. This final translation was then compared to the original to ensure equivalence and consistency, except for intentional changes involving culturally appropriate words like personal names, object names, contexts, and terminology. Any inconsistencies were resolved through discussion. This was used with the Chinese teachers in Guizhou only. All other teachers in the study were asked and answered the questions in English.

RESULTS

In this chapter, the results are discussed through techniques of comparison and contrast. In this way, the similarities and differences among teachers from Australia, the Mainland China, Hong Kong SAR, and the United States can be seen. Detailed findings about the beliefs of effective teachers of mathematics in each region can be found in Perry (2007), Wang and Cai (2007a, 2007b), and Wong (2007).

Teachers' Views about Mathematics

What is the nature of mathematics? Of the three fundamental questions investigated in this study, this one received the most varied responses among teachers from the four regions. Overall, teachers from Australia and U.S. both hold more to the functional view of mathematics, which focuses on its usage in the physical world. Teachers from the Chinese mainland and Hong Kong are of the Platonist view, meaning that they focus more on the internal structure of mathematical knowledge (Ernest, 1989). A discussion of some of the major similarities and differences across the four regions follows.

Mathematics is practical. Of all the common themes among the four regions, the utilitarian aspect of mathematics was dominant. There is agreement among teachers from Australia, the Chinese mainland, Hong Kong SAR, and the U.S. that mathematics is applicable to real life problems and that it is a necessary skill for living. All of the teachers interviewed from four regions felt that mathematics has many utilitarian aspects, including being applicable to other disciplines.

For Australian teachers, mathematics is *one of those essential subjects that allow us to function in the world.* (AU1)

For teachers from the Chinese mainland, mathematics is practical in daily life and can help people solve real life problems in an efficient way. It is *a science as well as a necessary tool for life.* (CH8)

For teachers from Hong Kong SAR, the practical significance of mathematics constituted a salient theme in the teachers' responses.

In daily life, [a] child may face problems in books. When they grow older, they use it in buying [a] house. I think that we learned some skills and methods of calculation, then apply them in life to solve problems continually. (HK11)

For U.S. teachers, mathematics could provide a new perspective for looking at the world:

I see it as a tool in order to solve problems. ... But it's a tool that enables people to do things or to reach goals that they have. The substance of mathematics would be things like a set of rules, a set of methods that allow me to achieve goals or achieve things I'm trying to do or other people are trying to do. (US5)

Mathematics is a language. Mathematics as a language was the second-most common theme relating to the nature of mathematics across the four groups of teachers. Though this belief was held more prevalently in some regions than others, what is meant by mathematics having the nature of a language is the same: it is a system of knowledge that provides the means of description and explanation of natural phenomenon. Yet, it is very different from commonly used languages like Chinese or English. The formal language of mathematics is a logical framework of rules and terms that can be used effectively to solve problems of many kinds and

communicate the problem-solving procedures to others in a somewhat universal dialect.

The emphasis on mathematics being a language decreased as the analysis moved through data from the Mainland China, Hong Kong SAR, the U.S., and Australia, respectively. It is possible that the description of mathematics as a language is held more strongly in the Mainland China and Hong Kong SAR because of the language's relation to the Platonist view of mathematics (i.e., language, being a structure itself, is related to the *structural view*). On the other hand, this view is held less by Australian and U.S. teachers because of their emphasis on the *functionality* of mathematics.

Mathematics is derived from real life. Only the teachers from the Mainland China explicitly argue that mathematics is derived from real life, though it is implicit in some of responses of Hong Kong SAR teachers. All of the nine teachers from the Mainland China believe that mathematics is an abstract and generalized knowledge system refined from real life problems.

Mathematics stems from real life ... but it is the knowledge refined [tilian] from real life. Once our ancestors help us get the knowledge, we can directly apply the general knowledge without considering some unnecessary features of each specific real life problem. (CH3)

This view extends the orientation of mathematics as practical that is held by the majority of teachers from all four regions.

Mathematical knowledge is abstract. This particular characteristic of mathematics drew out a sharp distinction among the four groups of teachers: specifically between the Eastern and Western regions. Teachers from the Mainland China and Hong Kong SAR had considerably more to say (both quantitatively and qualitatively) about the abstract nature of mathematics than did teachers from Australia and the U.S. There was a decreasing emphasis on the abstract nature of mathematics, in the following order: the Mainland China, Hong Kong SAR, Australia, and the U.S.

All nine of the teachers from the Mainland China differentiate mathematics knowledge from real life problems in that mathematics is an abstract and generalized knowledge system refined from real life problems. The real life problems provide the raw materials that can be purified and abstracted as mathematics knowledge.

The majority of the interviewed teachers from Hong Kong SAR said that developing abstract thinking in students is one of the objectives of teaching mathematics. Unlike teachers from the Mainland China, they did not give deep descriptions of what they thought abstraction is. However, they spoke mainly of the process of developing abstract, logical thinking in the classroom and, while they did not use the term, expressed this abstraction as a facet of mathematization (De Lange, 1996; Gravemeijer, Cobb, Bowers, & Whitenack, 2000).

In Australia and the U.S., few teachers explicitly report that mathematics is abstract. The reluctance to teach and encourage students to learn abstract principles

is evident, especially in the U.S. For the most part, that which is concrete is the focus in the classroom and lessons of Australia and the U.S.

This distinction between the Eastern and Western cultures in regards to the abstract nature of mathematics and how it affects mathematics education is predictable in light of the previously discussed views of the usefulness of mathematics. It follows from the Mainland China and Hong Kong SAR's structural or Platonic view that the teachers place greater importance on the abstraction of mathematics than the countries that hold more to a functional view.

Teachers' Views about Mathematics Learning

To discuss teachers' views about mathematics learning, we focus on the three main themes derived from Cai (2007):

- The nature of understanding – This includes the teacher's belief of what understanding is and how the teacher should help the students gain understanding of mathematical ideas.
- Memorization and understanding – What role, if any, does memorization play in a student's development of understanding? Should memorization come before or after understanding?
- The role of practice – What role does practice play and how much is necessary? What kind of practice develops understanding?

The Nature of understanding. In terms of the importance of understanding, teachers from the four regions all agree that understanding is the ultimate goal of learning mathematics and that using real-life problems and concrete experiences can facilitate mathematical understanding.

Understanding means being able to apply knowledge flexibly.

Teachers from the four regions agree that an indicator of mathematical understanding is the flexible application of what has been learned to problem situations that require the students to use what they have learned in different ways.

For understanding, I think the first step is they can accept the rule as a fact ... Second, when the rule appears in another format, s/he can still think in the reverse manner. I think this is understanding. (HK2)

[Understanding] is being able to use what you are able to apply in many different situations rather than just applying a skill or a piece of knowledge in one situation repeatedly. (AU12)

Another common theme in terms of the nature of understanding is that students are able to communicate what they have learned. When students are able to communicate with others using mathematical language, they display their understanding of the ideas being communicated. Teacher AU11 exemplifies this theme:

Understanding is achieved when they are able to explain the "why," the "how," and the "do" in a situation using mathematical language to support their explanation.

Understanding at concrete and abstract levels. Teachers from the four regions agree that mathematics understanding should start from students' concrete experiences. However, different regional groups of teachers have different views on how concrete examples should be used in mathematical learning. While the U.S. teachers put their emphasis on helping students realize the relatedness between mathematics and real life problems, teachers from the Mainland China tend to encourage students to derive abstract concepts and thinking from concrete examples. All of the teachers from the Mainland China argue that the ultimate goal of introducing concrete examples is to help students derive abstract mathematics concepts. Once the students' understanding reaches this abstract level, they can be freed from the constraints of concrete representations. After the students have established some abstract mathematical concepts, the teachers should emphasize the importance of connecting different concepts and integrating them into a systematic knowledge system. While most teachers from the Mainland China emphasize the importance of helping students master abstract and connected mathematics concepts, only one U.S. teacher (US7) explicitly mentions that helping students connect abstract concepts is important. Instead, most U.S. teachers express their reluctance to encourage students to learn mathematics on an abstract level (e.g., deriving formulas) especially in 6th or 7th grade classrooms. It seems that most of the U.S. teachers think their students at this stage are cognitively incapable of thinking abstractly. Teachers from the Mainland China thought differently.

While teachers from the Mainland China and the U.S. seem to hold quite different views about understanding at the concrete and abstract levels, teachers from Australia and Hong Kong SAR hold views between these extremes. For Australia and Hong Kong SAR teachers, while concrete experiences offer great opportunities for fostering mathematical understanding, the individual characteristics of the learner need to be taken into account before offering particular materials.

Some of the interviewed teachers from Australia and Hong Kong SAR explicitly point out that concrete materials are particularly relevant for certain groups of children (such as slow learners or students have missed out on learning a particular topic) and not for others (such as the more able or gifted learners). The level of abstraction teachers can expect students to reach depends on the characteristics of students and nature of the mathematics to be learned.

Memorization and Understanding. Though they all agree that memorization plays an important role in mathematical understanding, teachers from the four regions do not fully agree on what that role is or to what degree memorization is important. There was also an over-riding concern as to whether memorization should come before or after understanding. For teachers from the Mainland China and Hong Kong SAR, memorization can come before or after understanding. However, for Australia and U.S. teachers, memorization can only come after understanding. Nevertheless, memorization after understanding is held in higher regard than memorization before understanding (or rote memorization), though some of the

teachers from teachers in the Mainland China say that perhaps the latter could be an intermediate or transitional step towards understanding the mathematics.

While students start with rote memorization (without understanding), they should be able to gradually come to understanding by practicing. (CH8)

While teachers in the Mainland China express the value of memorization, they also make a distinction between what they call live knowledge and dead knowledge. Live knowledge is easy to transfer when solving new problems. In particular, CH1 uses the Chinese idiom *juyifansan* (knowing one concept and applying it into three situations⁴) to describe live knowledge. In contrast, dead knowledge cannot last long and is difficult to apply and transfer into a new situation. It seems that the teachers in the Mainland China believe that even that knowledge which is memorized before understanding must eventually be converted to live knowledge.

In general, teachers from Hong Kong SAR do not believe that memorization has a central role in mathematics learning.

Memorization may have some effect on mathematics learning, but it is not an important component. (HK4)

However, when asked what kind of memorization (if and when used) is best, a higher regard for memorization after understanding than for rote memorization was expressed. The majority of teachers in the Hong Kong SAR group view rote memorization as a final alternative for the student when she does not have understanding of that knowledge:

If there is something we really cannot understand, we should memorize it first as to tackle the examination. (HK9)

Though there is a general inclination among the teachers from Hong Kong not to make memorization (whether before or after understanding) an imperative, the teachers value the type of memorization that follows understanding and therefore makes the knowledge mentally available for application:

After the students understand, then memorization is important. It would be useful if s/he has a good memory. In fact, if s/he understands, memory would be useful to future application. (HK8)

Many of the Hong Kong SAR teachers were able to distinguish between memorization after understanding or even memorization and understanding enhanced in parallel. These teachers seemed to have a preference for the former type of memorization.

Many of the Australian teachers give high regard to memorization as the recall of pertinent information. They support the process of memorization as a key factor in learning almost as strongly as teachers from the Mainland China.

Memory is very important. They have to start off with a core amount of information. (AU2)

I think there's a place for memorization. I'm glad to see that the new syllabus puts some emphasis back on learning times tables. I think that is very important. Along with that comes understanding. (AU1)

However, Australian teachers do not necessarily believe that rote memorization can serve as a transitional step to understanding. The majority sees rote memorization – retaining facts without understanding – as something to be avoided. When the Australian teachers speak of memorization, they tend to add words such as “reinforce,” “connections,” and “understanding.” In general, the Australian teachers believe memorization should follow understanding.

The U.S. teachers are all in agreement that memorization after understanding is the type of memorization that is valuable. They believe it is necessary for retaining knowledge, applying the knowledge to solve problems, and learning new knowledge. However, when it comes to rote memorization, the interviewed teachers from the U.S. display a variety of views. Some believe it has little to no use, while others see it has something that is necessary.

I think that if they encounter something enough times, they're just going to remember it anyway. That rote memory is not something they are going to remember. (US8)

In contrast, US9 suggests:

I think some people that follow the NCTM standards⁵ very closely would disagree with me, but I still think there's a place for memorization and rote memorization of basic facts.

In summary, the idea of understanding before memorization seems to be the most prominent trend. The teachers interviewed from the Mainland China, Hong Kong SAR and Australia explicitly affirm that rote memorization can be useful in making knowledge quickly accessible and as a last resort for examination when understanding is not fully developed. Only teachers from the Mainland China expressed the view that rote memorization could also be useful as a transition to understanding.

Role of practice. All teachers from the four regions view practice as important, but to varying degrees. Teachers from the Mainland China place as much value on it as they did on memorization. Teachers from Hong Kong SAR, much like teachers from the Mainland China, view practice as a means to facilitate understanding, but some also present a minimalist view:

They don't need a lot of exercises, [just] one in each type. If you understand, you just have to have a quick glance [to understand] and don't need to do a lot. (HK7)

This minimalist view is also observed in U.S. responses such as:

I don't do a lot of practice ... students in my classroom don't necessarily get a lot of practice repeatedly on [a] particular concept. They may only be

exposed one or two times to a particular concept, and then we move on.
(US11)

One significant difference among teachers from the four regions is that teachers from the Mainland China were the only group who did not mention the risk of over-burdening the students with ‘drill and kill’ practice. Teachers from Hong Kong SAR, Australia, and the U.S. all shared concern that practice can be overdone and student’s interest in the subject can be dulled.

Yes they [exercises] are important, but I do not agree on letting students do the same kind of exercises too much unless there are variations. The worst case is that students do not think over the question after doing massive exercises. I do not agree with mechanical training. (HK5)

Teachers from Australia and the U.S. seemed to share similar concerns related to practice:

I think that you only need to go so far as realizing that most people are confident with the idea and not flogging it to death. (AU13)

I find that if you give too much, it’s like they’ll just turn off from it ... especially with word problems. You know, you don’t want to turn them off either. (US10)

Teachers’ Views about the Teacher and Teaching

While understanding teachers’ views about mathematics and learning of mathematics can provide an important context, the major focus of this study is to understand the teachers’ views about effective mathematics teaching. How a teacher decides to run a classroom is, to a great extent, a reflection of her/his goals for maximizing students’ learning and her/his beliefs and values in relation to the subject being taught. Hence, questions in this study about teachers’ views about mathematics teachers and mathematics teaching have elicited much core information.

Characteristics of an effective teacher of mathematics. The question of what characterizes an effective teacher of mathematics has accentuated the differences in the beliefs of teachers from the Eastern and Western regions. For example, teachers from Australia and the U.S. had much more to say about the teacher’s enthusiasm and rapport with the students than teachers from the Mainland China and Hong Kong SAR. On the other hand, teachers from the Mainland China and Hong Kong SAR focused on how well the teacher prepares and presents a lesson and the ability to provide clear explanations of the points to be covered in the lesson.

– Strong background in mathematics

Nearly all the teachers from Australia, the Mainland China, and Hong Kong SAR make a strong point of this characteristic of effective teachers of mathematics.

According to their statements, well-grounded knowledge and understanding of the subject is a crucial element in being able to effectively teach mathematics. In addition, teachers from the Mainland China place a very strong emphasis on understanding of the curriculum and the texts being used. According to teachers in the Mainland China, it is clear that an effective mathematics teacher should explore and study textbooks intensively and carefully and should try to predict the possibly difficult concepts for their students so that they can devise instructional strategies to overcome the difficulties. Teachers from Australia and Hong Kong SAR also emphasize deep understanding of the curriculum content and structure.

They have to have an understanding of the syllabus to start with and what they should be teaching. (AU5)

– *Adept in instructional skills*

All of the interviewed teachers agreed that an effective teacher should possess the skills needed to instruct properly. For teachers from Australia, this point was made implicitly through their discussion about making the lessons relevant to current society and balancing humor and authority. The following three specific instructional skills were mentioned consistently by teachers from the four regions:

Clear communication and explanation of the topic and goals (which requires their own knowledge of mathematics and of the curriculum).

Being able to use a variety of methods of instruction (manipulatives, thought-provoking lecture, etc.) according to the students' needs.

Building interest and maintaining it by varying methods and/or by making the topic, when possible, relevant to the students' experiences.

Although the teachers from all four regions agree that personal magnetism and solid mathematical understanding are both important traits of an effective math teacher, there is again a general difference between the teachers from the Mainland China and Hong Kong SAR, and teachers from Australia and the U.S. While teachers from the Mainland China and Hong Kong SAR highlight the need for the teacher's ability to provide insightful explanation and stimulate thinking, teachers from Australia and the U.S. focus more on how well the teacher can listen to their students and get them to interact with their teachers and one another.

– *Knowing and caring for the students*

The teachers from Australia, the Mainland China, and the U.S. all explicitly agree on the necessity of knowing and caring for their students. Hong Kong SAR teachers did not mention this point as a characteristic of an effective mathematics teacher. This does not suggest that these teachers do not believe that they should care for their students. However, it is clear that this point is not at the forefront of their thinking. The general theme among the teachers from Australia, the Mainland China, and the U.S. is that a teacher should understand the needs of the students and have the desire to understand their needs. For example, CH1 argues that a good teacher is always passionate in caring about students both in and out of the

classroom. CH2 further argues that this kind of passion not only builds a positive rapport between the teacher and students but also could directly impact on students' learning.

Overall, however, the teachers from Australia and the U.S. had more to say about building this positive rapport with the students than teachers from the Mainland China did.

Once there is a level of empathy with the student so that you know this person reasonably well, at least in terms of their interests, you can start to get somewhere. (AU9)

The [effective teachers] are caring. They relate to the students. ... I show them respect and they show me respect . . . (US10)

– *Classroom management*

Classroom management seemed to be much more important to the teachers interviewed from the U.S. than for any of the other three regions. One teacher asserted:

Well, first of all, I think the classroom management is truly important. If that's not there, we're not going to. . . nothing will be accomplished. (US2)

US5 also heavily emphasized this need:

In the public school, the number one thing that you have to have is you have to control the classroom. From my experience of teachers that appeared not to be as effective, it's more of a discipline issue and a control issue.

The teachers interviewed from the Mainland China did not mention anything about classroom management being a concern. This may reflect that, in most educational environments in the Mainland China, classroom management is simply not an issue. Neither does it appear to be an important issue with teachers from Hong Kong SAR or Australia, as nothing was said by the teachers from these two regions in regards to classroom management. It must be remembered that the interviewed teachers in all four regions were chosen for the study because they were effective mathematics teachers. Consequently, they would be expected to have solved issues around classroom management in their own teaching.

According to Ernest (1989), there are three basic teaching models related to the responsibilities that a teacher has in the classroom: facilitator model, explainer model, and instructor model. The first model, the facilitator, has the goal of having students develop confidence in establishing and solving problems. The explainer's aim is that the students gaining conceptual understanding with cohesive knowledge. Finally, the instructor's intention is that the student masters the skills necessary for proper performance. There are some teachers in the sample accessed in this study that fit more into one model than another. The description of an effective teacher according to teachers from the Mainland China and Hong Kong SAR seems to fit more into the instructor model. For teachers from Australia, it would seem that their description of an effective teacher is somewhat between the explainer and

facilitator: one who helps the students connect the knowledge mentally, yet encourages them to confront and solve mathematical problems themselves. The U.S. teachers' descriptions of an effective teacher generally seem to fit the facilitator model. This analysis is reinforced by the common belief among Eastern teachers that it is simply infeasible to facilitate individualized guidance with the large numbers of students present in their classrooms. These teachers believe that the role of classroom teaching is the transmission of knowledge: Ernest's explainer. It is often believed by these teachers that other facets of teaching/learning should be left to out-of-class hours.

Characteristics of an effective mathematics lesson: As might be expected, some of the aspects that were mentioned in regards to what makes a teacher effective were reiterated in the responses to the characteristics of an effective lesson, albeit from a different angle. There is a tendency for the teachers from the Eastern regions (the Mainland China and Hong Kong SAR) to emphasize the teacher-led aspects of the mathematics education in the classroom, while the Western region teachers (Australia and U.S.) emphasize the student-centered aspects (Leung, 2004). The possibilities of having a teacher-led yet student centered' classroom among Eastern regions should be noted (for details, please refer to Wong, 2004). It is noteworthy that Watkins (2008) even put forth the notion of learning center-ness and not just learner centeredness.

– *Active engagement of students*

All eleven U.S. teachers agree that active student engagement in the classroom is necessary to keep the students interested. Therefore, concrete examples are often implemented into the lesson:

I usually start off talking about why we're going to learn this topic, why we need this topic. Let's say with percent. And I have the kids say, well we need it for sales tax, to leave a tip at a restaurant ... so we talk about why we're dealing with this topic. Then I try to go into what does it really mean. (US7)

For the majority of the U.S. teachers, active student engagement also involves hands-on manipulative activities for the purpose of student exploration:

I think that investigation by students and allowing them to find rules, allowing them to find the way things behave is very effective compared to just always lecturing and giving formulas and telling them how things behave. (US5)

The teachers from Australia emphasize the students' verbal involvement more than the physical involvement as a characteristic of an effective lesson. There was little said specifically about the use of hands-on manipulatives. To them, active student engagement can arise by tapping the students' curiosity:

I think curiosity is a big thing with kids. . .and active student involvement. (AU2)

[An effective mathematics lesson is] one where all the conversation is about the maths, the students are engaged and there is not too much teacher talk. (AU13)

Such verbal engagement is also expressed as a respectful exchange between the student and teacher on what is being taught:

You try to make sure that children have the opportunity to question, to discuss, to answer and that there's an atmosphere where the children and teacher respect each other's views and that those are listened to. (AU1)

For teachers from Hong Kong SAR, student participation and involvement are the keys for understanding as well as achieving the learning objectives. Student participation is also the source of satisfaction in learning. For teachers from Hong Kong SAR, participation mainly refers to the vocalized interactions in classrooms.

Teachers from the Mainland China acknowledged that there was a necessity for a lively and comfortable learning environment. CH1 asserts that:

A terrible lesson is the teacher lecturing the whole lesson without student participation because you would have no idea whether your students understand the material.

In following this point made by CH1, all the teachers from the Mainland China seem to agree that concrete types of examples serve a purpose in helping the students understand mathematical concepts. However, there are varying opinions about how this should be practiced. For some teachers from the Mainland China, in order to understand a concept clearly, students should physically operate the concrete examples and tools. However, due to constraints of time and class size, other teachers argue that, in real teaching, a teacher often just demonstrates the process without having students manipulate tools.

It was also expressed by teachers from the Mainland China that the hands-on manipulatives should be used only for the objective of bringing about understanding, and, therefore, the teachers need to have the students contribute mentally and verbally to what they have just done physically.

– *Group activities/in-class student collaboration*

Significantly more was said about group activities and in-class student collaboration by the U.S. teachers than by those from Australia, the Mainland China, and Hong Kong SAR. Teachers from the Mainland China and Hong Kong SAR made no mention of this being a characteristic of an effective lesson. The teachers from Australia held a more middle-road view: small group activity was neither a necessity nor an impediment to an effective lesson, though the majority of Australian teachers interviewed utilize this pedagogical approach in their classrooms.

At times you need to have groupings. So you might have a core lesson and some work that the children who have obtained or [understood] that knowledge can go on with. Then you can spend some intense time with the ones that don't. (AU7)

For U.S. teachers, in-class peer interaction is essential to a lesson being effective.

And to have a problem like that where the kids are, you know, four of them together, are communicating mathematically, trying to solve a problem. Maybe not one of them individually could solve it, but all of them could solve it. (US3)

In general, it seems that the U.S. teachers are more comfortable with having group activities and discussion during the class time than are the teachers from the other three groups.

– *Coherence*

Of the four groups of teachers interviewed in this study, only teachers from the Mainland China and Hong Kong SAR explicitly addressed the issue of having a well-structured, coherent lesson for the class. All nine teachers from the Mainland China maintain that an effective lesson should coherently develop a well-planned content. The following statement is typical for teachers from the Mainland China:

An effective lesson should have all the steps [of instruction] closely serve the essential points...so that students can actively participate in each step. (CH3)

About half of the Hong Kong SAR teachers interviewed made mention that an effective math lesson is one which is well structured. For example:

One should think about what one is going to teach before a lesson, but should not pack too many objectives into a single lesson . . .another important point is the flow of the lesson [well-designed]. (HK5)

The teachers from Australia did not give as many specific responses that emphasized the need for coherence of a lesson as the teachers from the Mainland China and Hong Kong SAR. However, there was mention that lesson objectives should be made clear and that there needs to be a structured routine in the classroom:

I have clear goals to be reached, they know where the journey is going, it's very clear and they have to be focused. (AU4)

There were no comments about the coherence of lessons from the U.S. teachers interviewed.

– *Flexibility of teaching fits individual students' needs*

Teachers from Australia and the U.S. addressed the issue of flexibility with significantly more emphasis than did teachers from the Mainland China and Hong Kong SAR.

For the U.S. teachers, flexibility is a prominent characteristic of an effective mathematics lesson. However, it is primarily addressed through the characteristics of an effective mathematics teacher.

Being able to observe, and judge, and evaluate each student and meeting their individual needs probably is the most difficult and probably one of the most crucial parts [for an effective teacher]. (US4)

In regards to the math lesson itself, there is acknowledgment that the lesson needs to be appropriate to the students' stages of development.

[I]f you can gear a lesson so that it's just at the right spot for where the students are developmentally, where it stretches them just enough so they're not frustrated but it challenges them at the same time. (US9)

Australia's teachers were specific about how the lessons themselves should be both planned and yet flexible.

Most of my lessons are planned ... For example, you would have to have the resources you needed there, and if there is a child who needs concrete material, then you have to have it available. There has to be an ability to change. (AU7)

When new ideas have been discovered, when perhaps what I had planned is not what we've done at all, which is what happened to us this week because somebody came up with something and we've gone off on a tangent and discovered something totally new, then that is an excellent lesson. (AU3)

Some of the teachers from the Mainland China acknowledged the need for teaching flexibly in order to address students' needs. They agreed about the necessity of flexibility in the lesson, though that the flexibility is constrained both by the large number of students in the class and the amount of content that is required to be taught in a lesson. One teacher argues:

In terms of how to unfold a planned lesson, the teacher should always flexibly adjust his path according to student status. After a student answered a question, [I can find] what is still not understood by him. Then I will continue [to] explain it carefully. Therefore, I cannot just rigorously follow the plan. (CH2)

Only one teacher from Hong Kong SAR explicitly commented on the need for flexibility in order to be sensitive to the developmental pace of the students:

The teaching pace should be adjusted with the response of the students in the lesson. The teacher should not just blindly follow the lesson plan and let the lesson go on without considering students' response. (HK2)

– *Cultivating students' interests*

All four groups of teachers highlighted this characteristic of an effective mathematics lesson. For example, six teachers from Hong Kong SAR commented on the cultivation of student interest, giving various ideas on how this might be achieved.

Teaching aids, games, real-life examples, introducing various activities and outside readers to them [can help cultivate students' interests]. (HK8)

Other ways in which students' interest could be stimulated include drawing connections between different mathematical ideas in the syllabus.

They would feel surprised and this would initiate their thinking [too]. (HK7)

The importance of a teacher using a good question and answer technique in order to stimulate interest was canvassed by teachers from both Hong Kong SAR and the Mainland China.

[One has to] ask questions, from which can inspire students to further imagine ... [One should ask] how can we make use of questions to guide students to think something new, deeper, and those things they have never thought before. (HK1)

In order to broaden student participation, the teacher should design questions with different levels of difficulty to take care of good students, average students, and slow students. Therefore, in my class, I never invite one student to answer a question more than three times so that more students can have chances to answer questions. (CH2)

Many of the Australian teachers believe that cultivating interest in students is important. Some of these teachers point out that if one is able to begin the lesson in an interest-capturing way, then the student's interest is more likely to be maintained for the rest of the lesson.

When it is time for math groups it should be "Yes! Off we go to maths" and they should be coming into the classroom excited. For all sort of reasons, the mathematics classroom should be a place where they feel really good about themselves, where they're feeling really enthused to be there ... Not everybody feels like that all the time but there are times when the recess bell has gone and I am shooing them out the door and they're still not going. (AU3)

I like to start the lesson off with something that makes the children think. It doesn't have to be anything to do with the particular topic that you're learning but it just means that you are trying to get the answer to something. (AU13)

There is no specific comment made by teachers from the U.S. on the cultivation of student interest being a characteristic of an effective lesson. However, other comments reported above from U.S. teachers suggest that they see such cultivation as important. The majority of teachers from the Mainland China seem to agree. For example, cultivation of student interest can lead to ongoing motivation to learn mathematics.

At the beginning, they can learn some mathematics, and then they are willing to learn more mathematics. Finally, they enjoy mathematics. (CH8)

REFLECTION AND DISCUSSION

Mathematics impacts the way we understand our environment, control our finances, construct enterprises, and conduct business. Effective mathematics teaching is clearly of the utmost importance in developing the mathematical skills, knowledge and understanding required to enable this impact to be realized. Effective mathematics teaching requires effective mathematics teachers. The beliefs of such teachers about mathematics, mathematics teaching and mathematics learning are critical to ensure that there is sufficient mathematical capital being developed (Leder et al., 2002; McLeod, 1992). Hence, hearing the perspectives of teachers from a variety of regions about the effective teaching is invaluable.

The study reported in this chapter provides a cross-cultural (East compared with West) perspective on teachers' beliefs about effective mathematics teaching. Australia, the Mainland China, Hong Kong SAR, and the U.S. were selected for the study because they represent a spectrum of Eastern and Western cultures.

Though one may argue that beliefs constitute only one aspect of teachers' professional expertise, the link with teachers' teaching practices has been acknowledged (Furinghetti, 1998; Leder et al., 2002; McLeod, 1992; Pehkonen & Törner, 1998). The words of the 18th century Chinese scholar, Yuan Mei, that "*Knowledge is like the bow, ability like an arrow; but it is wisdom which directs the arrow to bull's eye*" (Siu, Siu, & Wong, 1993, p. 223) is pertinent to our thinking that teachers' beliefs about mathematics and about how mathematics should be learned and taught will influence their teaching practice.

Nature of Mathematics

Some of the beliefs of teachers from Australia, the Mainland China, Hong Kong SAR, and the U.S. about the nature of mathematics, and the learning and teaching of mathematics showed an East/West cultural dichotomy while others resulted in much more of an East/West cultural continuum. For example, the teachers from the Mainland China and Hong Kong SAR view the nature of mathematics from a Platonic view: their focus is on the internal, logical structure of mathematics, which reflects mathematics as an abstract body of knowledge. In contrast, the teachers from Australia and the U.S. place much emphasis on the functional view of mathematics: mathematics is a useful tool that is utilised everyday to solve real-life problems. For example, teachers from Australia and the U.S. placed more emphasis on mathematics being a language by which physical phenomenon can be described and explained than did those from the Mainland China and Hong Kong SAR. This does not mean that there is no acknowledgment by teachers from the Mainland China and Hong Kong SAR of the usefulness of mathematics in helping to solve real life problems. Rather, there is not as much emphasis placed on its functionality by teachers from the Mainland China and Hong Kong SAR as there is by teachers from Australia and the U.S.

Understanding, Memorization, and Practice

In regards to the nature of understanding, there was not a great deal of variance among the four groups of teachers. Teachers from the four regions by and large agreed that the goal of mathematics education is that the students gain understanding of the mathematics being taught. They also agreed that both the student's ability to apply the mathematics to various problems and her/his ability to communicate the learned mathematics to the teacher or other students indicate the presence of understanding. However, there is significant difference among the four groups of teachers in terms of their descriptions of the relationship between understanding and memorization.

Two types of memorization were identified: memorization *before* understanding (sometimes designated rote memorization) and memorization *after* understanding. For teachers from the Mainland China and Hong Kong SAR, memorization can come before or after understanding. However, for Australian and U.S. teachers, memorization can only come after understanding. For teachers from the Mainland China, memorization before understanding could serve as an intermediate step towards understanding; in other words, as long as this type of memorization leads to understanding, then the memorized knowledge is not simply dead knowledge. In general, however, teachers from all four regions agreed that understanding after memorization is ideal, though it is also acknowledged by those teachers that this is not always the case in the classroom. It seems that memorization is regarded as a means and understanding as the goal, though automation (memorizing by heart) is also regarded as important, especially when one needs to solve mathematics problems fluently (Kerkman & Siegel, 1997; Wong, 2006). Other research supports the hypothesis that the excellent academic performance of Asian learners on international mathematical comparison programs may be due to a synthesis of memorizing and understanding which is not commonly found in Western students (Marton et al., 1996; Marton et al., 1997; Watkins, 1996).

In terms of the role of practice, it seems that the teachers from the Mainland China are the most comfortable ones with having students practice since there was no concern expressed about over doing it; there were only indications of its value. Hong Kong SAR teachers place nearly as high a value on practice as teachers from the Mainland China do, but only if that practice is constituted as exercises with variations. Australian and U.S. teachers, in general, are not as committed as the Mainland China and Hong Kong SAR teachers are to the value of practice. Teachers from Australia and the U.S. shared a common concern that practice can be overdone and, therefore, dull student interest. None of the teachers from the Mainland China, however, expressed this concern.

Characteristics of an Effective Teacher

Teachers from the Mainland China, Hong Kong SAR, and Australia agree that competence in mathematics is a necessary characteristic of an effective teacher. It was also stated (especially by teachers from the Mainland China) that a teacher's in-depth understanding of the curriculum and textbooks is key for an effective

teacher. The U.S. teachers did not note this as an important point in their responses.

Teachers from all regions concurred that a teacher should both understand the needs of her/his students and have an interest in catering to these needs. Teachers from the U.S. believed that effective teachers need to have appropriate classroom management skills, particularly in terms of discipline and control. Teachers from the other three regions did not address this issue, possibly because classroom discipline does not seem to be as major an issue with them, particularly for teachers from the Mainland China and Hong Kong SAR. The general difference here was that the teachers from the Mainland China and Hong Kong SAR emphasize the ability of a teacher to provide the information with clarity and to stimulate thinking, while the teachers from Australia and the U.S. emphasize the ability of the teachers to listen to the students and to get them to respond with interest.

Characteristics of an Effective Lesson

Based on the responses of the teachers from the four regions, it appears that the teachers from Australia and the U.S. are more comfortable with frequent use of hands-on manipulatives than teachers from the Mainland China and Hong Kong SAR are. Hong Kong SAR teachers, in particular, suggested that if physical manipulatives are used, they are commonly used by the teacher for purposes of demonstration and not by the students, mainly because of time restraints. The teachers from the Mainland China tend to stress verbal engagement over physical engagement on the part of the students. The teachers from the Mainland China and Hong Kong SAR did not mention in-class group activities, yet this is stated as a characteristic of an effective lesson by many of the U.S. teachers. In fact, group activities are usually included in U.S. teachers' lesson plans, but not in those from Mainland Chinese teachers (Cai, 2005; Cai & Wang, 2006). Teachers from Australia saw group activity neither as a necessity nor an impediment to mathematical understanding. U.S. teachers tended to focus on students' engagement and interaction during mathematics lessons while teachers from the Mainland China and Hong Kong SAR emphasized the importance of coherence of a lesson. In summary, what do teachers consider characterizes an effective lesson? Teachers from the East tend to have more of a teacher-led view of classroom instruction than do teachers from the West, who hold more of a student-centered view.

It is impractical to look for a national/regional script of mathematics teaching. Yet classroom practices are often shaped by cultural, environmental, and societal assumptions. Watkins and Biggs (2001) have warned that teaching and learning traditions that appear to work well in a certain culture may not necessarily work in another. For instance, when a high-stake examination is the ticket to the future, fast and accurate solutions to mathematics problems are needed. The ramifications of such high-stake examinations are often manifested through parental expectations and their impacts on classroom practices. When there are larger class sizes, such as is the case in both the Mainland China and Hong Kong SAR, hands-on explora-

tions can become difficult and individual care is often left to after-class hours (Gao & Watkins, 2001; Wong, 2004).

In this study, we see a broad-stroke linkage between teachers' beliefs in mathematics, their image of an effective mathematics lesson, and that of the effective teacher. For instance, for the two Eastern regions (the Mainland China and Hong Kong SAR), since the mathematics teachers generally hold a Platonic view, mathematics knowledge and structure are stressed in the teachers' responses about effective mathematics teaching. It is important to let the student understand the generalization (Cai, 2004). Consequently, practice plays a central role. Though these teachers fully understand the importance of individual guidance, cultural norms such as class size and current practices seem to have determined that this can only be done after class (Gao & Watkins, 2001; Wong, 2004). For these teachers, the major task of classroom teaching lies in the transmission of knowledge. The teacher must be well prepared and have the lesson well structured, so as to run a teacher-led, yet student-centered mathematics lesson. As Watkins (2008) pointed out, the focus should be put on learning, or more precisely, the tripartite interactions among teacher, student and learning. Two things are prerequisites: thorough understanding by the teacher of the curriculum and textbook and the establishment of a classroom routine by the students and teacher. This may explain partially why classroom management is not a major concern among Eastern mathematics teachers. Students are accustomed to the various routines in the flow of classroom teaching—when to talk, when to do seat work, when to open one's book, when to look at the chalk-board (or computer projection), and so on early at an early age (Wong, 2004)—students know the routines and implement them. In the two Western regions, there is a much stronger emphasis both on student-centered approaches to mathematics teaching and learning and on the need for the mathematics being learned to be practical and relevant to the learners. While teachers' understanding of the mathematics being taught is seen as important, reliance on planning, knowledge of the syllabus and textbooks is not as strongly emphasized as is the case with the teachers from the Mainland China and Hong Kong SAR. Both U.S. and Australian teachers see that part of their being effective teachers relies on their knowledge of the students and their understanding of the students' needs. Consequently, the functional understanding of mathematics leads to less structured lessons that are more able to reflect flexibly the needs of the students.

In this study, we have confined our investigations to teachers' perspectives on the effectiveness of mathematics teaching. Many similarities and some differences have been discerned across the four regions considered. The qualitative methodology used in our research and the small numbers in our regional samples could be seen as a limitation in our study. Further investigation is needed to see if these similarities and differences are sustainable across the populations concerned.

There are many other international studies that take different starting points than those of the current study, and they have found results that both contrast and comparison with those presented in this chapter (Clarke & Keitel, 2006; Fan et al., 2004; Leung et al., 2006; Stigler & Hiebert, 1999). It is possible that asking different questions of different people in different ways might result in different

conclusions. What is notable are the similarities that have resulted from these different approaches.

On the other hand, long term research on the CHC (Confucian Heritage Culture) learning phenomena has shifted its attention from looking for cultural attributes for the success of Asian learners to identifying good practices in both Eastern and Western cultures (Wong, 2004). More needs to be done to uncover what is happening every day in the mathematics classrooms of different regions. Detailed investigations in mathematics classrooms need to be continued. Classroom observations, interviews, telling of teachers' and students' life-stories, among many other methodologies could be used (Clarke & Keitel, 2006). The perspectives of pre-service and novice teachers and school students need to be canvassed further in a consistent, valid cross-cultural methodology. We need to strive for as full a picture as possible of effectiveness in mathematics teaching and learning so that future generations of students in all regions can benefit. The current study is but one approach that seems to have proved fruitful results.

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NOTES

- ¹ Though there had been extensive discussions on the distinctions among terminologies like "conception," "belief," "view," "image," ... (see, e.g., Pehkonen, 1998; Philipp, 2007), in this chapter, we used them quite interchangeably.
- ² Special Administrative Region
- ³ Since Hong Kong SAR is not a country, we use "region" throughout this chapter to designate the four jurisdictions in which the study was conducted.
- ⁴ Words of Confucius quoted in Chapter 1 of the Analects.
- ⁵ The mathematics curriculum and evaluation standards published by the National Council of Teachers of Mathematics of the U.S. in 1989 and 2000.

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APPENDIX: DETAILS OF PARTICIPANTS

Table 1. Details of Australia participants

<i>Code</i>	<i>Gender</i>	<i>Qualifications</i>	<i>Years of teaching</i>	<i>Other relevant data</i>
AU1	M	DipTeach	30+	Fellow of College of Teachers, London; Fellow of Royal Geographic Society; Excellence in Teaching award from Australian College of Education; Former Head of Mathematics/Deputy Headmaster of prestigious independent school
AU2	F	BTeach	11	School art coordinator; Count Me In Too ¹ training
AU3	F	DipTeach	20	Professional development in gifted and talented education and systemic mathematics program
AU4	F	B.Ed; GradCert (Productive Pedagogy)	20	Extensive Count Me In Too training; Former mathematics consultant; Taught in England and Ecuador
AU5	F	B.Ed	25	Training and experience in special education; Former mathematics consultant
AU6	F	B.Ed	32	Conference presenter
AU7	M	B.Ed	30	Extensive Count Me In Too training; Assistant Principal
AU8	F	DipTeach	20+	National Literacy and Numeracy Award; Former mathematics consultant; Extensive Count Me In Too training
AU9	M	B.Ed	25+	Council Member, Mathematical Association of Victoria; School Numeracy Coordinator
AU10	F	B.Ed	28+	Conference presenter; School Numeracy Coordinator
AU11	F	B.Ed, GradCert (Teaching)	18	School Numeracy Coordinator
AU12	M	B.Ed, M.Ed	19	Taught in Canada for 8 years
AU13	F	B.A, GradDip (Primary)	20	Taught Spanish in primary schools for 13 years

¹ Count Me In Too is a systemic numeracy program introduced into the majority of New South Wales government schools.

Table 2. Details of participants from the Mainland China

<i>Code</i>	<i>Gender</i>	<i>Qualifications</i>	<i>Years of teaching</i>	<i>Other relevant data</i>
CH1	F	Graduate from a Normal School	19	Active participant of an instructional improvement project
CH2	F	Graduate from a Normal School, Received a bachelor's degree through advanced training	25	Frequent contributor to books or teaching journals for teachers Gave model lessons
CH3	F	Graduate from a Normal School, Advanced training in mathematics and mathematics education	22	Active participant of an instructional improvement project Gave many model lessons for other teachers
CH4	F	Graduate from a Normal School, Advanced training in mathematics and mathematics education	30	Gave model lessons for other teachers Wrote articles for a local teaching journal
CH5	F	Graduate from a Normal University, took additional mathematics courses in another Normal University	19	Became a teacher researcher two years ago, she does not teach in class, but helps other teachers to teach
CH6	M	Graduate from a Normal School, took additional mathematics courses from a University	23	Active participant of an instructional improvement project
CH7	M	Graduate from a Normal School, took courses from another University and received a bachelor's degree	34	Has been a teacher researcher for 11 years, he does not teach in class, but helps other teachers to teach Gave many model lessons for other teachers
CH8	F	Graduate from a Normal School, was taking classes from a University	21	Has been a teacher researcher for three years Received a prize for a teaching competition
CH9	F	Graduate from a Normal School, was taking additional courses in math and math education courses from a Normal University	20	Gave model lessons for other teachers Wrote articles for a local teaching journal

Table 3. Details of Hong Kong participants

<i>Code</i>	<i>Gender</i>	<i>Qualifications</i>	<i>Years of teaching</i>	<i>Other relevant data</i>
HK1	M	Teacher Cert., B.Ed.	17	Member of the editorial board of a local mathematics education periodical
HK2	F	Teacher Cert., B.Ed.	6	Council member of a local mathematics education professional body served at the government Education Department as seconded teacher for a year
HK3	F	Teacher Cert., B.Ed.	10+	Council member of a local mathematics education professional body
HK4	F	Teacher Cert., B.Ed., M.Ed.	10	Council member of a local mathematics education professional body
HK5	F	Teacher Cert.	25	Teaching practice supervisor of a university
HK6	M	B.S., PGDE	5	Head mathematics teacher in school
HK7	F	Teacher Cert.	22	Curriculum leader in school Team member of a university project on students' motivation of learning
HK8	M	Teacher Cert., B.Ed.	15	Principal member of the government's Curriculum Development Council (mathematics)
HK9	F	Teacher Cert., B.Ed.	15	Head mathematics teacher in school team member of a university project on students' motivation of learning
HK10	F	Teacher Cert., B.Ed., M.Ed.	17	Member of the government's Curriculum Development Council (mathematics)
HK11	M	B.S., PGDE	10+	Senior teacher in school
HK12	F	B.Ed., M.Ed.	12	Member of the editorial board of a local mathematics education periodical teaching practice supervisor of a university

Table 4. Details of U.S. participants

<i>Code</i>	<i>Gender</i>	<i>Qualifications</i>	<i>Years of teaching</i>	<i>Other relevant data</i>
US1	F	B.A.; M.Ed.; Received National Board Certification	12	Led workshops for other mathematics teachers; Attended mathematics education conferences or workshops regularly
US2	F	B.S., Completed 18-credit in-service training	5	Received the Outstanding First Year Teacher award; Member of teacher leadership team for two years; Attended mathematics education conferences or workshops regularly
US3	F	B.S.; Completed 25-credit in-service training	9	Received the Outstanding First Year Teacher award; Member of teacher leadership team for four years; Led workshops; Attended mathematics education conferences or workshops regularly
US4	F	B.A.; Received National Board Certification	14	Led workshops for other mathematics teachers; Lead teacher at summer enrichment academy; Attended national and regional mathematics education conferences or workshops regularly
US5	M	B.A.; Six credits away from receiving a M.Ed.	11	Participated in a math education project; Led workshops for other mathematics teachers; Attended national and regional mathematics education conferences regularly
US6	F	B.A.	11	Received Presidential Award for Excellence in Science and Mathematics Teaching Led workshops; Participated in several math education projects; Attended national and regional mathematics education conferences or workshops regularly

US7	M	B.S.; M.Ed.; and Ed.D.	32	Participated in several math education projects; Led workshops; Attended mathematics education conferences or workshops regularly
US8	F	B.A.; Enrolled in a M.Ed. program	14	Led workshops; Participated in several math education projects; Attended mathematics education conferences or workshops regularly
US9	M	B.A.; M.Ed.	10	Led workshops; Participated in several math education projects; Attended mathematics education conferences or workshops regularly
US10	F	B.A.	23	Led workshops; Mathematics Department Chair; Attended national and regional mathematics education conferences or workshops regularly
US11	F	B.S.	7	Led workshops; Participated in several math education projects; Attended mathematics education conferences or workshops regularly

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WHAT IS EFFECTIVE MATHEMATICS TEACHING? INTERNATIONAL EDUCATORS' JUDGMENTS OF MATHEMATICS LESSONS FROM THE TIMSS 1999 VIDEO STUDY

It has been well documented that classroom mathematics teaching differs across countries (Clarke et al., 2006a; Givvin et al., 2005; Hiebert et al., 2003a, 2003b; LeTendre et al., 2001; Leung, 1995; Stigler & Hiebert, 1999). Much less is known about whether leading educators in different countries differ in their views about what kinds of teaching *should be* occurring. Do the differences in classroom practices across countries mirror differences in experts' views or do the differences exist in spite of shared views among experts? The goal of this chapter is to address this question. Specifically, we address whether there is variability in the vision of mathematics educators in five different countries¹ with respect to what constitutes effective practice.

There is, of course, a growing literature on what constitutes effective mathematics teaching. Recent summaries of the research (e.g., Hiebert & Grouws, 2007; National Research Council, 2001; Reynolds & Muijs, 1999) note that what counts as "effective" depends on a variety of factors, including the particular learning goals of interest. For example, one set of instructional practices has been shown to be effective for helping students develop quick execution of skills whereas another set of practices has been shown to be effective for helping students develop conceptual understanding. Other factors influencing one's view of "effective" include what role one plays in the educational system. Teachers' views can differ, at least in emphasis, from those of researchers and policy makers (Wilson, Cooney, & Stinson, 2005). We raise these issues simply to make it clear that the aim of this chapter is *not* to contribute directly to this literature on what should count as effective teaching but rather to describe differences and similarities in views that currently exist among mathematics educators. That is, we do not aim to promote a particular perspective on effective teaching or to advocate for a particular form of instruction but rather to enrich our understanding of how mathematics educators in different countries evaluate teaching with respect to its effectiveness.

Our central question is whether mathematics educators in different countries judge the effectiveness of teaching differently and, if so, in what ways. There are reasons to believe that mathematics educators across countries could make similar judgments with regard to effectiveness. That is, mathematics educators in different

countries might share visions of effective classroom practice. The increasing communication among international educators with international audiences through journals, international handbooks and other publications, and international associations and meetings, provide opportunities for collaboration and discussion around issues of mathematics teaching and learning. Common ideas and recommendations might be developed and adopted across countries. In addition, international comparisons (of the kind represented in this chapter) could themselves lead to increasingly shared views among mathematics educators (Clarke et al., 2006b). Even the process of constructing and using a shared achievement instrument for international comparisons imposes some level of homogeneity across countries (Keitel & Kilpatrick, 1999). These activities are more likely, of course, to influence the views of mathematics educators involved in these joint activities than of classroom teachers more removed from international communications.

It is also possible to formulate plausible hypotheses for why mathematics educators across countries might differ in their views of effective teaching. Variations in visions could emerge from the same kinds of differences in cultural traditions that have been proposed as explanations for differences in teaching itself (Stigler & Hiebert, 1999). Leung (1995, 2001), Cai, Perry, and Wong (2007), and Tweed and Lehman (2002) noted differences between Eastern and Western cultures that might affect educators' views of effective teaching. In particular, Leung (1995, 2001) suggests that Eastern values could lead to greater emphases on mathematical content and teacher-directed practices whereas Western values could promote greater emphases on the processes of learning and student-centered practices. We interpret Leung's argument to open the possibility that deep cultural differences, of varying kinds, might lead to differing views across countries in what counts as effective mathematics teaching.

No single study can answer definitively the question of whether, and to what degree, mathematics educators across countries share views of effective teaching. But we believe the study we report begins to address the question with a unique set of empirical results. The study was enabled by the Third International Mathematics and Science Study (TIMSS) 1999 Video Study. The Video Study provided a common set of videotaped eighth-grade mathematics lessons that mathematics educators could evaluate. A shared set of concrete referents reduces the implicit differences that often exist in verbal descriptions of teaching in the abstract. By asking the mathematics educators to watch a common set of lessons, we had a common basis on which to compare educators' views of mathematics teaching.

This chapter describes the judgments of mathematics education groups drawn from most of the participating countries in the TIMSS 1999 Video Study and provides comparative information about their views of teaching. Do they focus on similar or different features when evaluating mathematics teaching? Do they identify similar or different features when they compare what they see with what they believe to be effective mathematics teaching?

METHOD

Sample

The countries that participated in the mathematics portion of the TIMSS 1999 Video Study were Australia, the Czech Republic, Hong Kong SAR, Japan,² the Netherlands, Switzerland, and the United States. Random, nationally-representative samples of eighth-grade mathematics lessons were videotaped in each of these countries. All of these countries, except the United States, are considered to be high achieving in eighth-grade mathematics, based on their scores on the TIMSS 1995 mathematics achievement test (Beaton et al., 1996; Gonzales et al., 2000), which was used to select countries for inclusion in the TIMSS 1999 Video Study. For more information about the sample of lessons collected for the TIMSS 1999 Video Study, see Hiebert et al. (2003a).

Groups of mathematics educators from five of the seven countries that participated in the TIMSS 1999 Video Study participated in this study: Australia, the Czech Republic, Hong Kong SAR, Switzerland, and the United States.³ The educators were selected by the country's National Research Coordinator, who served as the country liaison for the TIMSS 1999 Video Study.⁴ The educators were experienced mathematics teacher educators and/or researchers, and typically were on the faculty of prominent universities in their country or researchers at national research institutes.⁵ Aside from the North American educators (who were all members of the TIMSS 1999 Video Study steering committee), and the National Research Coordinators, the educators in the other countries did not have a particular connection to the Video Study. In each country, three to nine mathematics educators participated in the meetings. The National Research Coordinator joined the discussion at times to offer opinions of the lessons or otherwise guide the nature of the discussions. It should be noted that no group of that size can adequately represent the perspectives shared across an entire nation. Small numbers of educators may, even within one culture, come to different judgments. Therefore, the size of the educator groups should be kept in mind when interpreting results.

Procedures

Selecting the lessons. The data for this study were the educators' judgments of lessons drawn from six of the seven countries. New lessons from Japan were not collected for the 1999 Video Study, so educators from Japan were not part of the international research team and Japanese lessons were not viewed during this study.

Because our goal was to compare judgments of the lessons (rather than the lessons themselves), it was critical that all groups of educators viewed the same set of lessons. We asked each country's team of coders⁶ to select from the full set of lessons up to five lessons that captured the common features of teaching in each

country. The code developers and the director of the mathematics portion of the TIMSS 1999 Video Study reviewed the lessons and chose two from each country for the educators to view.⁷ The exception to this was Switzerland, from which three lessons were chosen (i.e., one from each language area: French, German, and Italian).

Selecting two (or three) lessons that capture the common features of teaching in each of the six countries is, of course, nearly impossible. No single lesson, or pair of lessons, represents teaching in a country. But it is possible to select lessons that reveal some practices commonly seen across the full data set within a country and to avoid lessons that are clear outliers. Taken together, the selected lessons displayed some of the variation apparent in the teaching practices across the six countries and served as a common referent for the educators' comments about effective teaching.

Analyzing the videotaped lessons. Meetings of each group of educators were held in each country. During these meetings the educators watched and discussed the selected lessons from each country. Switzerland hosted three mathematics educator groups, one in each language region. Each of the Swiss groups is treated separately in this report.

The mathematics educator groups were told to watch the set of lessons from each country in whatever order they preferred. They were asked to focus on the *methods* used to teach mathematics—in particular, the kinds of learning opportunities provided for the students and the nature of the mathematical reasoning that was evident by the teacher or the students. The task was quite open-ended, and comments on other dimensions of similarity and difference deemed especially striking were invited. The written task given to all participants is included as Table A1.

The groups were asked to devote approximately two hours to each of the six countries, watching and discussing the designated lessons. A written report based on these discussions was requested, and thus the groups were asked to keep notes during their meetings. It was suggested that the written report indicate the dimensions that the group members used to compare the lessons, along with comments about each country's lessons along those dimensions. In addition, the groups were asked to summarize their conclusions regarding major similarities and differences among the lessons.

The mathematics educator meetings were conducted in 2001 or 2002, for two or three day periods. In each country, the meetings were hosted by the National Research Coordinator of the TIMSS 1999 Video Study from that country. At least one member of the TIMSS 1999 Video Study mathematics code development team was also present to assist in organizational and technical details. The math educators viewed the 13 lessons over the computer, using specially designed software that allowed them to hear the lesson in its native language, and see a running English transcript.

RESULTS

Each group of mathematics educators wrote a report summarizing their judgments about the videotaped lessons they viewed.⁸ The specific organization and nature of those reports was not heavily dictated beforehand, and they varied substantially. Some groups wrote comments on each lesson whereas other groups combined their comments for the set of lessons from a country. Some groups had more to say about certain lessons or countries than others. By request, the submitted reports represented the consensus views of all members of the group.

Summarizing the comments from each mathematics educator group took place in several stages. First, at least two of the authors of this chapter reviewed the comments made by each educator group. They examined comments from each country in turn, keeping a written record of those that applied to all lessons in a country. When comments within an educator group differed across the lessons within a given country, the authors noted that as well. Next, the authors discussed the results of their review, considering issues of inclusion as well as language. In particular, the authors considered whether all common notions were captured and adequately supported by the data, and whether the language they used to summarize the content of the educators' comments was as close as possible to the original language (or its English translation). Discussion continued until consensus was reached. Finally, the comments were loosely classified as addressing the role of the teacher, the role of the students, the mathematics content, or the climate of the lessons. These four classifications emerged from the data and served as a means of organizing the data without theoretical bias. There is considerable overlap between these categories and they are not intended as reliable distinctions. Rather they serve to help organize the array of data and are intended to be a useful device for the reader. The result of this process appears in the Appendix as Tables A2 through A7.

It is important to keep in mind that the original data (i.e., individual educators' comments on the lessons) have undergone multiple phases of aggregation. Individual educators' comments formed the basis for discussion among the members of the educator groups, the result of which was recorded by one of the group members, in some cases translated into English, and then interpreted and further aggregated by our research team. It is certainly possible that successive aggregation made subtle intercultural differences less visible, which may lead to the incorrect assumption that each country has a unified voice. To help accurately convey the perspectives of each educator group, we maintain their language as much as possible and frequently provide direct quotations from their reports.

In the sections that follow, we present a summary of the judgments offered by the educators about each country's lessons, in turn. We then look across the comments of the different lessons offered by each country's group and attempt to characterize the primary concerns voiced by each country. These two ways of aggregating and juxtaposing educators' comments begin to address the question of whether mathematics educators from different countries hold similar or different views of effective mathematics teaching.

Comments on the Australian Lessons

As seen in Table A2, most of the mathematics educator groups had a good deal to say about the two Australian lessons. There was a high level of agreement among the groups, with the exception of the Australian educators' comments about the role of students.

Teacher's role. All of the mathematics educator groups felt that the Australian teachers played a strong, guiding role. They characterized the lessons as teacher-"directed," "led," or "guided."

Students' role. With the exception of the Australian educators, all of the mathematics educator groups described the Australian students as having a rather small and undemanding role. For example, the Czech educators stated that the students "show little activity." The Swiss-Italians maintained that "the students participate in a guided activity that does not present any meaning to their learning." And, the Swiss-French educators wrote that "not much is asked of [students]... [The] teacher seems resigned to ask a minimum of intellectual effort from his students." Similarly, the North Americans noted that the teachers made things easy for the students. They wrote that the "teacher seems to protect students from thinking." By contrast, the Australian educators felt that the videotaped teachers attempted to involve their students through classroom activities.

Content. There was widespread agreement that the mathematics content in the Australian lessons was at a low level and too heavily focused on procedures or rules, with not enough attention to mathematical concepts and reasoning. Specifically, the group from Hong Kong SAR referred to the lessons as having "superficial mathematics content" and as being "rule-oriented." The Swiss-German educators claimed that "the development takes place on a purely procedural level, without an in-depth understanding of the mathematical concepts that stand behind those procedures." The Swiss-French educators referred to the lessons as being "very algorithmic." Even the Australian educators agreed that the two lessons were "sometimes unfocused, so significant features may be unclear at the end."

A number of the mathematics educator groups (including the Australian, Czech, Swiss-French, and Swiss-German groups) felt strongly that the teachers did not place a strong enough emphasis on the correct use of mathematical language. The Czech educators noted that "the covered material is focused on practical application, which sometimes leads to inaccuracy in terminology." The Swiss-German group noted that there was a "low level of accuracy in defining mathematical terminology." Their Swiss-French colleagues provided a more elaborated comment, stating that "we have the feeling that the level of expectation has been lowered to negotiate the students' participation. This translated into the teacher's tendency to use 'easy' terms for the students. He ends up speaking without rigor and with limited correctness." The Australians offered another explanation, stating that there was a "deliberate de-formalizing of maths as though teachers acknowledged that it was unpalatable to students."

Three of the educator groups pointed out that the mathematics content was contextualized. Specifically, the Australians described “definite attempts to involve students and show relevance.” The Czechs described the content as “based on application” and the North Americans used the word “situated.”

Climate. Most of the groups commented on the climate in the Australian lessons and agreed that it was informal. However the Hong Kong SAR educators felt the “laissez-faire” atmosphere led to a lack of discipline, while the Swiss-Germans felt that the classes were “relaxed” and “well-disciplined.”

Comments on the Czech Lessons

The Czech Republic lessons provoked a relatively large number of comments by most of the educator groups. There was general consistency among their impressions, particularly with respect to the content (see Table A3). The educators tended to speak positively about the content, although they had more mixed impressions regarding the role of the students.

Teacher’s role. The mathematics educator groups agreed that the Czech teachers played a dominant role in their classrooms. They used words such as “authority” “dominated,” “directed,” and “controls” in their descriptions of the Czech teachers and their behavior. However, the North Americans added that the teachers did not “shoulder the entire workload.”

Students’ role. There was some discrepancy among the educator groups regarding the extent of the students’ participation in the Czech lessons. Most of the groups noted that the students demonstrated their knowledge, especially when they were publicly examined. On the other hand, several described the Czech students as having few opportunities for choice and rarely asking questions. The educators from Hong Kong SAR wrote that among the participating countries, students in the Czech Republic had the lowest degree of choice in the mathematics they were to learn. The Swiss-German educators commented that “Czech students learn by reproducing the cognitive structure that has been presented by the teacher ... There is hardly any leeway for alternative solution methods.” Interestingly, the Czech educators described the students as “passive.” On the other hand, a number of educator groups pointed out that high expectations or demands were placed on the Czech students. The Swiss-French described this mixture of roles as “responsibility without autonomy.”

Content. The educator groups largely shared positive impressions regarding the mathematics content in the Czech lessons. Words used to describe the content included “demanding,” “difficult,” “dense,” and “rich.” Several groups mentioned that there was an emphasis on rules and procedures as well as concepts and processes. For instance, the Swiss-German group wrote that “next to the ‘how’ of the procedures also the ‘why’ of the procedures has central meaning.” Both the

North American and Swiss-German educators discussed their positive impression that the content was developed in a logical, linear fashion, with the Swiss-Germans stating that “the content is taught in small steps and meaningful units that build on each other.” The Australians, North Americans, and Swiss-Germans noted the use of correct mathematical language.

Climate. Comments on the climate in the Czech lessons were largely related to formality. The educators described the atmosphere as “formal,” “serious,” “respectful,” and “very intense, focused, and disciplined.” Two of the groups (Australian and Czech) mentioned that there was a conspicuous distance between the teacher and students.

Comments on the Hong Kong SAR Lessons

Many of the comments regarding the Hong Kong SAR lessons were similar in nature to those made about the Czech Republic lessons. However, there was somewhat less agreement among the educator groups regarding the nature of the mathematics content in the Hong Kong SAR lessons, as seen in Table A4. Specifically, the groups varied with respect to how much emphasis they felt was placed on reasoning as opposed to computation.

Teacher’s role. The mathematics educator groups all agreed that the Hong Kong SAR lessons were dominated or highly directed by the teacher. Several groups noted that the teachers provided much of the information to their students. For example, the Czechs referred to the lesson as being “conducted as a lecture” and the Swiss-Germans felt that the “teacher walks through [the material] step by step in a clear and direct way.” The group from Hong Kong SAR expressed the opinion that “knowledge was given by the teacher.”

Students’ role. There was a shared sense that the Hong Kong SAR students did what they were told but were not active participants in the lessons. For example, some groups noted that although the Hong Kong SAR students answered the teachers’ questions or worked at the board, their involvement still appeared to be minimal. The Australian group wrote that students were “absolutely quiet and submissive” and the Swiss-Germans wrote that student participation was limited to very short, predictable comments or statements. Similarly, the Swiss-Italians wrote that “one does not see a willingness to invent the occasions for the students to discover the learning process themselves.” Even the Hong Kong SAR educators agreed and declared that the “students seem to be treated as calculation and computation machines.” The Hong Kong SAR and North American educator groups both described the demand placed on the students as moderate. Specifically, the North American group wrote that “students were not expected to struggle ... but they were expected to think. To engage.”

Content. Opinions about the nature of the content varied. Some groups felt that the difficulty level was high (the Swiss-French) and that reasoning or deep understanding of concepts was encouraged (the Australians and Swiss-Germans). However, the Hong Kong SAR and Swiss-Italian educators were more critical and saw the lessons as too focused on teaching technical abilities and rules, with not enough emphasis on mathematical concepts.

The Australians and Swiss-Germans noted the use of correct mathematical language in Hong Kong SAR lessons. In addition, the Australian and North American groups discussed the important role played by the textbook in guiding the structure of the lessons and providing examples. Specifically, the North American educators wrote that “both of the lessons were based on examples worked out in the students’ book.”

Climate. The climate in the Hong Kong SAR mathematics lessons was perceived as “disciplined,” and “formal.” The Australians commented that there was a “sense that the work was serious and important.” Most groups felt that the students worked hard, and the Swiss-German and Swiss-Italian groups described the students as learning to be “competitive.” Three of the educator groups (the Australians, Czechs, and Swiss-Italians) mentioned that there was a noticeable distance between the teacher and students.

Comments on the Dutch Lessons

The mathematics educator groups had mixed impressions of the Dutch lessons with respect to all four areas considered – teacher’s role, students’ roles, content, and climate (see Table A5). Most of the groups described both positive and negative qualities of the lessons, but the Swiss-Italians were especially critical.

Teacher’s role. Most of the mathematics educator groups felt that the teacher’s role in the Dutch lessons was largely as a guide or partner, rather than directing or dominating the lesson. For example, the Swiss-Germans said that “teachers in the Netherlands play the role of facilitators, whereas in all the other countries teachers try at least for part of the time to transmit or demonstrate knowledge.” And, although the Czech educators called the lessons “frontal” and teacher directed, they also described a “partner-like attitude of the teacher toward the student.”

Students’ role. Comments on the role of the students in the Dutch lessons were varied. Two of the educator groups (the Czechs and Swiss-Germans) noted that the Dutch students were afforded a large degree of responsibility and control over their own learning and pointed out that the students worked with their peers. For example, the Swiss-German educators commented that “during seatwork phases the students have a high degree of control. The students solve the problems independently, sometimes without any prior introduction or knowledge.” At the

same time, three of the educator groups (the Australian, Hong Kong SAR, and Swiss-Italian educators) felt that the Dutch students were not involved enough in the public portions of the lessons. The Australian educators felt that there was “little evidence of student understanding or involvement.”

Content. Most of the educator groups agreed that the content in the Dutch lessons was not particularly demanding, and saw little emphasis placed on reasoning and higher order thinking. For example, the Hong Kong SAR educators felt that although the lessons covered many mathematical problems, mathematical concepts were not explored. The Australians referred to the lessons as “procedure-bound.”

Although some groups were largely critical of the content in the Dutch lessons (the Swiss-Italians felt that the lessons were “without any real occasions to learn”), other educators had more positive judgments. Specifically, the North Americans commented that the “ideas are sophisticated for eighth graders” and that the topics appeared to be mathematically challenging. The Swiss-German educators noted that the Dutch lesson style teaches students how to solve problems by exploring on their own, stating that “the Dutch students are self-proficient learners who explore procedures and who are supposed to construct their own mathematical understanding.”

Several groups discussed the dominant roles played by homework (the Czechs and Swiss-Germans) and the textbook (the Australians and North Americans) during Dutch lessons. These groups described the textbook as setting boundaries on the content presented and helping to determine the learning structure. The North American group, for instance, stated that “the book has the role of giving a set of problems for students to work and in the working the mathematics will be developed.”

Climate. Mathematics educators described the climate in the Dutch lessons as “casual,” “informal,” “laissez-faire,” and “permissive.” Some groups (Hong Kong SAR, Swiss-Germans, and Swiss-Italians) believed that these qualities were problematic, and further described the lessons as lacking discipline and rapport between the teachers and students. Specifically, the Hong Kong SAR group called the lesson “out of control” and the Swiss-Italians claimed that “the teacher finds it difficult to relate to the students. Therefore there is not a real educational rapport.” The educators from the Czech Republic theorized that “Dutch liberalism is recognizable in the schools. The informal and even partner-like relationship between the teacher and a student during instruction reflects this liberal attitude.”

Comments on the Swiss Lessons

The number of comments included in Table A6 regarding the Swiss lessons is somewhat fewer than that of the other countries. This is largely due to the fact that the mathematics educator groups had varying impressions of the three Swiss lessons they watched. Some of the groups, particularly the North American and Swiss-French, made only a few comments that extended to all three lessons.

However, there was a good deal of consistency across the educator groups with respect to their comments that did apply to all of the Swiss lessons.

Teacher's role. Most of the mathematics educator groups felt that the Swiss teachers played a strong role, and two described the lessons as “teacher-led.” For example, the Swiss-German educators noted that “knowledge is often developed in a teacher-led instructional conversation. The teachers show significant effort to provide opportunities for discovery and individual problem solving.” On the other hand, the Swiss-French educators perceived the teachers as providing “indirect or subtle guidance.” The lessons were also praised as “well prepared,” “well planned,” or “highly structured” by the Czech, Hong Kong SAR, and Swiss-German educator groups, respectively.

Students' role. There was general approval among the mathematics educator groups in terms of the roles played by the Swiss students. For example, the Czech and Swiss-German educators noted that students actively participated in “problem solving.” The Swiss-German and Swiss-Italian groups saw some opportunities for the students to discover and “construct cognitive steps independently.” The Hong Kong SAR educators felt that there was “quite high demand on students' work” and “a lot of student involvement” whereas the North American educators felt that the degree of thinking students were asked to do ranged across the three lessons from low to moderate.

Content. Mixed opinions were expressed regarding the difficulty of the mathematics content in the Swiss lessons. Some of the educator groups (Australian, Hong Kong SAR, Swiss-German educators) felt that there was evidence of mathematical thinking and reasoning, and opportunities for students to gain a deeper understanding of the material. For instance, the Australians commented that “in different ways all three lessons encouraged mathematical thinking.” The Swiss-German educators wrote that “teachers often address with their comments and questions the ‘why’ of mathematical steps” and that there is “an attempt on the side of the teachers to trigger constructive thinking processes rather than receptive ones.” However, the Swiss-Italian educators felt that “the educational objectives are different” across the three lessons, and the Czech educators described the content as “little, but well connected.”

Climate. Comments on the climate in the Swiss lessons were quite positive. The educator groups described the lessons as “enjoyable,” “respectful,” and “serene,” and consisting of “motivated” students. The Czech educators found that a “friendly relationship between the teacher and students is prominent.”

Comments on the U.S. Lessons

Most of the mathematics educator groups had a considerable amount to say regarding the U.S. lessons, especially regarding the students' role and the nature of

the mathematics content (see Table A7). Their impressions, particularly with respect to these two dimensions, were generally consistent and largely negative.

Teacher's role. The mathematics educator groups agreed that the U.S. teachers played a strong role, and guided the students step-by-step through the lessons. As the Swiss-Germans explained, "the teachers demonstrate the content to be learned in front of the whole class." The Czechs wrote that the "teacher has control over the lesson."

The North American educators observed that a main role of the teachers in the videotapes appeared to be helping their students prepare for tests. Specifically, they said of one lesson that "the entire period was devoted to the upcoming quiz ... [The teacher] was comprehensive in his coverage of all that would be on the quiz." The Czech and Swiss-German groups commented that the assessments served as a motivating factor for the students. The former group wrote that "students use [the teacher's] presence to learn material they do not know, so that they can succeed in taking a test."

Students' role. Almost all of the groups mentioned that students' involvement in these lessons was relatively infrequent and/or of limited depth. Specifically, the Swiss-German group wrote that "verbal participation of students is infrequent and limited. Often these are answers to 'fill-in-the-blank' type questions, which focus on the immediately following step." The Hong Kong SAR educators noted that the "students [were] not expected to think deeply" and were given "low-level tasks only." The Swiss-French educators noted that "to participate, students don't need a great intellectual implication." Similar concerns raised by the North American and Swiss-Italian educators were that students did not engage in processes that involved discovery or reasoning. The Swiss-Italians wrote that "the lessons are based on knowledge already known to the students. One does not see a willingness to invent the occasions for students to discover the learning process themselves." The North Americans commented that "no reasoning was asked of the kids."

Content. The content in the U.S. lessons was deemed to be lacking and was widely criticized. The educator group from Hong Kong SAR described "computation and factual recall. Superficial math content (content is 'slim')." Other educator groups described the lessons as being too focused on procedures and rote learning. Missing was any justification of techniques involved, attention to relational understanding, and learning about higher order mathematical principles. Educators from the Czech Republic and North America used the words "content: little" and "not very demanding," respectively. The Swiss-German educators wrote that "there is a great redundancy of content."

The North American educators noted that the content in the lessons seemed constrained by what was in the textbook or other published materials. They said that "the text plays the dominant role. The teacher uses problems and review sheets that are provided by the text" and "the teacher added no value to the problems

provided by the text ... The teacher just blindly follows the text without clueing the kids into how this fits with what they have been doing or where they are going.” The role of the textbook was not brought up by any of the other groups.

Climate. The educators described the climate in the U.S. lessons in mostly positive terms such as a mixture of an “easy,” “respectful” relationship between the teacher and the students. The group from Hong Kong SAR noted that “order was kept.”

Characteristics of Educators’ Comments

To get a more direct sense of the similarities and differences among the views of the educator groups with regard to effective mathematics teaching, we looked at the recurring themes within the comments of each mathematics educator group. What features of teaching did the educators in, say, Australia focus on and how do these features compare to those most frequently voiced by the mathematics educators in the other countries? By looking across Tables A2-A7, we abstracted those features of lessons that were frequently identified by each group. Table 1 contains our interpretation of these themes. Readers are encouraged to review Tables A2-A7 and check our interpretation against their own. Because classroom climate is such a culturally-bound and difficult construct to interpret and summarize, we included only the teacher’s role, the students’ role, and the mathematics content in Table 1.

It is apparent to us that all of the mathematics educator groups shared concern about two issues: who is doing the mathematical work during the lesson (teacher and/or students) and how demanding or challenging is the content. Concern with who does the mathematical work is evident by the nearly unanimous focus on the level of dominance or guidance with respect to the teacher’s role and an almost equally frequent focus on the involvement of students. The demand of the content was sometimes voiced with respect to the mathematical nature of the content (e.g., that it requires deep, conceptual understanding) and sometimes with respect to the expectations for students (e.g., that students be actively engaged in reasoning and in presenting their thinking).

At a general level, there was some indication that most educator groups agreed not only on the importance of these two features of teaching, but on the way in which these features define effective teaching. With regard to who does the mathematical work, the groups indicated that students must participate in doing some of the work and that teachers frequently do too much of the work. Teachers do too much by demonstrating a procedure step-by-step and leaving students to practice, by controlling the discussion too tightly, by not providing students opportunities to reason about and make sense of the material, and so on. With regard to content, most groups expressed concern that the content in at least some lessons was not challenging enough, and that there was little mathematically to learn.

Table 1. Major Themes of Educator Groups' Comments

Educator Group	Teacher's role	Students' role	Content
Australia	– Level of dominance or guidance	– Degree of involvement (what are students doing, in what activities are they engaging)	– Nature and amount of reasoning required – Degree of structure and coherent sequencing (including use of textbook toward this end) – Use of technical language
Czech Republic	– Level of dominance or guidance	– Degree of involvement (are students active or passive)	– Level of demand – Degree, nature of structure in activities
Hong Kong SAR	– Level of dominance or guidance – Degree of apparent planning	– Degree of involvement (do students present their ideas) – Level of demand/expectations – Student choice in mathematics/activities – Degree of collaborative work	– Richness vs. shallowness – Degree of conceptual thinking vs. rules/computation
North America	– Level of dominance or guidance – Degree to which textbook is followed	– Level of demand/expectations	– Degree of focus on rules – Degree of structure determined by text
Swiss-French	– Level of dominance or guidance	– Degree of involvement (are students asked to do mathematical work and to do problems at the board) – Degree of responsibility and autonomy	– Level of demand – Nature and amount of reasoning required – Use of technical language
Swiss-German	– Level of dominance or guidance (control)	– Degree of involvement (are students asked to provide independent constructions) – Degree of student choice – Degree of cooperative work	– Extent of conceptual development – Use of technical language
Swiss-Italian	<i>Varied, no prominent theme</i>	– Degree of involvement (are students asked to provide independent constructions)	– Extent of learning opportunities (how does content allow students to learn) – Degree of focus on technical competence – Clarity of lesson objectives

Behind this general agreement, there were some interesting differences in how these constructs were defined or at least what aspects of these constructs were emphasized. Student involvement, for example, meant somewhat different things to different groups of educators. Some mathematics educators (e.g., Australia) focused on involvement through participation in lesson activity, often in solving real-life problems. Other mathematics educators (e.g., Swiss-German, Swiss-Italian) spoke about involvement as opportunities to construct ideas independently. The extent to which students can choose how they participate was also mentioned by some groups (e.g., Hong Kong SAR, Swiss-German), as was the degree of responsibility and autonomy held by students (Swiss-Italian).

A second example of differences in nuance behind more general agreement can be found in the groups' commentaries about the mathematics content and its level of demand. The educators in Hong Kong SAR, for example, expressed a clear separation in the richness of content from whether it is mostly about rules versus concepts. That is, some lessons were judged to be rich mathematically *and* mostly about rules or procedures. In these cases, the Hong Kong SAR group looked for the ways in which the rules were developed. This same disaggregation between richness versus shallowness from rules versus concepts was not apparent in all groups.

All of the educator groups indicated that many of the lessons did not meet their standard of effective teaching. In fact, several groups commented that none of the videotapes depicted the type of teaching they would like to see in eighth-grade mathematics classes. Although these comments do not mean that all groups held exactly the same vision of effective teaching, it does suggest that they set a similar level of standard for what counts as effective.

CONCLUSIONS

Shared Images of Effective Teaching

At a general level, the mathematics educators from all countries—Australia, Czech Republic, Hong Kong SAR, North America, and Switzerland—tended to agree on the features they were looking for in effective mathematics teaching. First, all groups indicated that students should play a significant role in the classroom rather than having it dominated by the teacher. They agreed that the teacher should allow students to actively participate in making sense of the mathematics. Second, all of the groups commented on the degree of challenge posed by the mathematics content in the lessons. Those lessons deemed mathematically demanding were held in greater esteem than those whose content was considered slim or minimal. Also noted by the educators was the extent to which the content was developed. They reacted negatively to lessons that simply demonstrated rules and procedures and asked students to practice.

It is interesting that mathematics content was identified and described so often by the groups, given that the task description encouraged the participants to focus on instructional methods rather than content (see Table A1). Clearly, all mathematics educator groups viewed content as an essential aspect of methods—as a main ingredient in defining effective mathematics teaching.

Differences Among the Educator Groups

Differences among the educator groups appeared to revolve around the ways they defined some of the general constructs, such as student involvement, as well as the degree to which they judged a desired attribute, was present in the videotaped lessons. In the case of student involvement, the Swiss-German and Swiss-Italian educator groups looked for evidence that students provided independent constructions. The Hong Kong SAR and Swiss-French educators looked for students to present their ideas and work. The Swiss-French were interested also in whether students were asked to *do* mathematical work. The Australian educators took a broader view, examining the activities in which students were engaged, and the Czechs were more general still, noting whether students were active or passive.

In many cases the educators agreed on the qualities they wanted to see, but differed in their interpretations of how prominently those qualities were displayed in the particular lessons they viewed. For instance, with regard to the mathematical content in the Hong Kong SAR lessons, all of the educators wanted to see evidence of reasoning and conceptual thinking. However, some of the groups more than others were confident that these features were present in the lessons.

On a few occasions the educator groups agreed on the presence (or absence) of lesson features, but disagreed about whether these features were positive or negative. This was particularly true with respect to their comments about lesson climate. For example, all of the educator groups who commented on the climate in the Dutch lessons agreed that it was casual and informal, yet two of the groups saw the relaxed atmosphere in a positive light, whereas three of the groups felt that it led to discipline problems.

Occasionally, the mathematics educators viewed the lessons from their own country differently from their international colleagues. For example, most of the educator groups described the two videotaped Australian teachers as playing a strong, guiding role, while leaving the students relatively uninvolved in demanding mathematics. By contrast, the Australian educators felt these two teachers did not assume a dominant role and did attempt to involve their students, although they acknowledged that some of their attempts might have failed. Each educator group's frame of reference is likely shaped by their experiences working with many teachers within their country. The Australian educators' comments might have been made in relation to other Australian teachers they have worked with and observed. Indeed, these different frames of reference for the various educator groups underscore the significance of the fact that, on the whole, the educators' impressions of lessons from their home country were in agreement with the impressions of the educators from (at least some of) the other countries.

What Does This Tell Us?

Before returning to the question of whether educators across countries share a vision of effective mathematics teaching, we need to insert an important caveat. We have been implying, by naming the mathematics educator groups with the countries in

which they work, that a group represents the views of an entire country. This cannot be true because surely there is greater variation among mathematics educators within a country than represented by the small number of group members who participated in this study and, further, we have no way of knowing how representative of the country are the views of the group members. Having said this, we do believe that the National Research Coordinators in each country invited participants who were credible mathematics educators in each country. In addition, there was no apparent connection among the participants across countries. Consequently, the results can be interpreted as judgments of respected mathematics educators in each country arrived at independently of the judgments of those in other countries.

Given this context, we believe it is fair to conclude that the mathematics educators across the five countries in this study share some significant components of a vision of effective teaching. To address the opening question of the chapter, the data we have presented suggest that the differences in classroom teaching across countries (as analyzed and reported elsewhere; see Hiebert et al., 2003a, 2003b) do not result from translating into practice different visions of effective teaching promoted by mathematics educators in the respective countries. In general, the visions of effective teaching expressed by mathematics educators across countries show much more similarity than do the country's classroom practices. In other words, many of the differences in classroom practices across countries appear to exist in spite of experts' views rather than because of them. As noted in the introduction, this might be due to the much greater level of communication and collaboration across country boundaries by mathematics educators than by classroom teachers.

Reinforcing this conclusion is the fact that all the mathematics educators participating in this study expressed some disappointment in the quality of classroom practices revealed on the videotapes, including lessons in their own country. In their judgment, their visions of effective teaching were not being realized in the classroom. All of these countries share the problem of translating visions into practice. Recall that the countries selected for inclusion in the TIMSS 1999 Video Study, with the exception of the United States, had high mathematics achievement as measured by the TIMSS 1995 assessment. Therefore, it is particularly interesting that the examples of teaching from these countries did not meet the standard set by this group of international educators.

There is a sense in which the findings on educators' judgments parallel our earlier work on classroom practices. At a general level of description, classroom practices look quite similar; at a more specific level, there are interesting and educationally significant differences (Givvin et al., 2005; Hiebert et al., 2003b). It might be fair to say the same thing about educators' judgments. Mathematics educators in different countries identified common, key features of effective instruction, but defined some of the general constructs in different ways. We believe these differences could be the basis for productive international discussions among mathematics educators. We see the forums for international communications and collaborations not only as possible explanations for the current similarities of views but also as forums for continued discussions about differences. We conjecture that the learning opportunities of such

discussions will increase if they are centered on concrete referents of teaching. Imagine, for example, the learning opportunities for the mathematics educators who participated in this study if we could have arranged a joint conference of all participants to share and discuss their judgments of the videotaped lessons. Visions of effective teaching in each country clearly could be enriched through such collective examinations.

NOTES

- ¹ For convenience, in this chapter Hong Kong SAR is referred to as a country. Hong Kong is a Special Administrative Region (SAR) of the People's Republic of China. We held separate meetings with educators in each of the language regions of Switzerland, but here count Switzerland as a single country.
- ² The Japanese lessons collected for the TIMSS 1995 Video Study were reanalyzed for the TIMSS 1999 Video Study.
- ³ A meeting of Dutch mathematics educators was planned but did not take place due to logistical difficulties. Because Japanese mathematics educators did not participate in the 1999 data collection, they were not part of this study. We will refer to the group that met in the United States as North American; 4 of the members were from the United States and 1 was from Canada.
- ⁴ The National Research Coordinators for the TIMSS 1999 Video Study were: Australia – Jan Lokan (1998-2001) and Barry McCrae (2002-2003), Australian Council for Educational Research; Czech Republic – Jana Strakova, formerly at the Institute for Information on Education; Hong Kong SAR – Frederick Leung, The University of Hong Kong; Switzerland – Kurt Reusser, University of Zurich; and United States – Patrick Gonzales, National Center for Education Statistics.
- ⁵ In the case of the Czech Republic, the National Research Coordinator selected experienced mathematics teachers to participate.
- ⁶ An international group of specially trained coders analyzed the TIMSS 1999 Video Study data. They were fluently bilingual and could therefore watch the lessons in their original language. In most cases, they were born and raised in the country whose lessons they coded (Jacobs et al., 2003).
- ⁷ With the exception of one Hong Kong SAR classroom, the lessons viewed by the mathematics educators are not the same as those released publicly for the mathematics component of the TIMSS 1999 Video Study. Most of the lessons viewed by the mathematics educators are considered “restricted use,” meaning that permissions were not obtained for these lessons to be shown publicly. However, copies of the lessons for public release, either in their entirety or as a series of clips, are available from <http://www.pearsonachievementsolutions.com/bkstore/index.cfm?action=dsr>.
- ⁸ Reports written in a language other than English were translated into English.

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APPENDIX

*Table A1. The Task Given to Each Educator Group**Task Goal*

The goal of the task is to provide an overall assessment of some key similarities and differences in eighth-grade mathematics teaching among the six participating countries based on viewing two “typical” lessons from each country. We would like you to focus on the methods used to teach mathematics rather than other features, such as the level of the content. In particular, we are interested in the kinds of learning opportunities provided for the students and the nature of the mathematical reasoning that is evident. However, we also welcome your comments on other dimensions of similarity and difference that you find especially striking.

Task Description

We would like you to spend about two hours per country--watching the lessons and discussing the nature of teaching that you see. We have edited the lessons so that you can view both lessons in about an hour; however, you are free to view the entire lesson if you would like. Please prepare a consensus written report (indicating any areas where there is disagreement among yourselves) that addresses the goal of the task described above. Don't worry about the written form of the report. Unpolished statements that capture your main impressions are fine. Examples are helpful.

Suggested format for your report. To increase the likelihood that meaningful comparisons can be made among the reports from each country, we suggest that you organize your comments in the following way.

1. Identify the dimensions you used to compare the lessons and provide all of the comments you made about each country's lessons along these dimensions (some dimensions might be relevant only for some countries). This requires keeping notes of your discussions while reviewing the lessons.
2. Summarize your conclusions about major similarities and differences among the lessons.
3. Suggest the major “stories” that you think can be told from these lessons (assuming the remaining lessons in the sample are similar to these).
4. Provide whatever comments you would like about the lessons from this set that you think are high quality or exemplary and identify the key elements that distinguish high quality lessons from the rest.

Table A2. Educator Groups' Impressions of the Australian Lessons

Educator Group	Teacher's role	Students' role	Content	Climate
Australia	<ul style="list-style-type: none"> Guide students to understanding 	<ul style="list-style-type: none"> Attempts to involve students through activities Little emphasis on public performance of students 	<ul style="list-style-type: none"> Deformalizing the math and showing its relevance Sometimes unfocused, so significant features may be unclear at the end Sequence and structure of the math not emphasized Reasoning not directly encouraged Technical language underused 	<ul style="list-style-type: none"> Casual and informal Many social interactions
Czech Republic	<ul style="list-style-type: none"> Teacher directed ("frontal") No clear lesson structure 	<ul style="list-style-type: none"> Little activity or involvement 	<ul style="list-style-type: none"> Not very demanding Inaccurate terminology sometimes used Based on practical applications 	<ul style="list-style-type: none"> Teacher is "natural authority" Informal attitude of students toward teacher
Hong Kong SAR	<ul style="list-style-type: none"> Teacher-led Not well planned 	<ul style="list-style-type: none"> Few hands-on activities No collaborative work No student presentation Relatively low expectations of students 	<ul style="list-style-type: none"> Low-level computation Superficial (shallow) math content Rule-oriented 	<ul style="list-style-type: none"> Laissez-faire atmosphere Lack of discipline
North America	<ul style="list-style-type: none"> Teacher helps students figure out the rules, makes things easy for them, is ally of the students, prepares them for tests Follows the spirit of the textbook, but not totally bound to it 	<ul style="list-style-type: none"> Students are protected from thinking very hard Low math demand on students 	<ul style="list-style-type: none"> Focus is on rules and procedures rather than concepts and conceptual understanding The math is situated 	<p><i>Opinions across the lessons were mixed</i></p>

Educator Group	Teacher's role	Students' role	Content	Climate
Swiss-French	<ul style="list-style-type: none"> - Teacher provides strong guidance, intervenes when there are errors - "Transmissive" – little autonomy given to students - Authoritarian, directed by the teacher 	<ul style="list-style-type: none"> - Little is asked of students, low expectations of students 	<ul style="list-style-type: none"> - Fragmented, low-level - Little emphasis on mathematically correct language 	<ul style="list-style-type: none"> - Amicable
Swiss-German	<ul style="list-style-type: none"> - Teacher takes control of the cognitive process and leaves little leeway in making independent steps - Teacher shows the correct solution or solution method in response to mistakes 	<ul style="list-style-type: none"> - Students expected to repeat and automate procedures demonstrated by the teacher in small steps - No opportunity to discover or construct math structures - Opportunities for cooperation between students not observed 	<ul style="list-style-type: none"> - Purely procedural, without an in-depth understanding of the math concepts that are behind the procedures - Lower emphasis on correct mathematical terminology - Terms are defined, developed, and then applied – a deductive instructional pattern 	<ul style="list-style-type: none"> - Relaxed but well disciplined - Sympathetic and respectful teacher-student relationship - Motivation is extrinsic – to produce correct solutions on the test
Swiss-Italian	<ul style="list-style-type: none"> - Completely teacher guided 	<ul style="list-style-type: none"> - Students learn by repetition, not by math reasoning - Don't create their own spontaneous knowledge 	<ul style="list-style-type: none"> - Very minimal content - Great attention to application of rules - No real learning opportunities 	<p><i>No comments made</i></p>

Table A3. Educator Groups' Impressions of the Czech Lessons

Educator Group	Teacher's role	Students' role	Content	Climate
Australia	<ul style="list-style-type: none"> Teacher is dominant 	<ul style="list-style-type: none"> Lesson is for teaching Learning through student activity or practice is separate or elsewhere Student's publicly assessed in beginning 	<ul style="list-style-type: none"> Appropriate technical language 	<ul style="list-style-type: none"> Little social interaction between teacher-students and students-students
Czech Republic	<ul style="list-style-type: none"> Teacher directed ("frontal") and dominated Lesson highly structured/ordered/routinized 	<ul style="list-style-type: none"> Passive Some use of groups 	<ul style="list-style-type: none"> Demanding Formal material covered 	<ul style="list-style-type: none"> Formal and authoritative Teacher keeps a distance from students
Hong Kong SAR	<ul style="list-style-type: none"> Teacher-led Continuous formative assessment Teacher has authoritative role 	<ul style="list-style-type: none"> A lot of student presentation Little student choice of the mathematics to learn High expectations of students 	<ul style="list-style-type: none"> Math content dense and rich Emphasis on computation and factual recall Both process and the correct solution are stressed 	<ul style="list-style-type: none"> Clear classroom routines Serious atmosphere
North America	<ul style="list-style-type: none"> Lessons carefully planned Teacher explores ideas beyond what happened in the book Teacher does not shoulder the entire workload 	<ul style="list-style-type: none"> Held accountable for learning Expected to do some learning on their own High math demand on students Students examined at the beginning 	<ul style="list-style-type: none"> Focus is on rules, technical aspects of math language, and symbol manipulation Linear organization to the math development, learning one idea or skill in preparation for the next 	<ul style="list-style-type: none"> <i>No comments made</i>
Swiss-French	<ul style="list-style-type: none"> Teacher is authority Activity guided by the teacher 	<ul style="list-style-type: none"> Students are involved Their work is important Students used as "technical assistants" and take part in the teaching workload 	<ul style="list-style-type: none"> High level of difficulty 	<ul style="list-style-type: none"> Students pay attention, have respect for the teacher

Educator Group	Teacher's role	Students' role	Content	Climate
Swiss-German	<ul style="list-style-type: none"> - Teacher organizes the lesson linearly and controls every detail of the instruction, even during private time - Through direct teaching, teacher makes the deep structure of the math procedures transparent - Lessons carefully planned and organized efficiently 	<ul style="list-style-type: none"> - Given responsibility without autonomy (i.e. rarely ask questions) - Students are responsible for doing a problem on the board; they get judged by their peers and graded by the teacher - Students reproduce the cognitive structure demonstrated by the teacher, with little leeway for alternative solution methods - Students' mistakes are exhibited publicly and recognized as learning opportunities - Phases of seatwork, but cooperation is prohibited - Students are given public feedback on their performance in the form of grades 	<ul style="list-style-type: none"> - Content is composed of small steps and meaningful units that build on each other - Knowing the "how" and "why" of procedures is important - Correct mathematical language is valued 	<ul style="list-style-type: none"> - Intense, focused, and disciplined atmosphere - High degree of pressure put on students - Emphasis on social comparison
Swiss-Italian	<ul style="list-style-type: none"> - Teacher concerned about the students' learning process 	<ul style="list-style-type: none"> - The math is carried through dialogue - Students don't ask questions but have an active role doing "research" and inductive thinking 	<ul style="list-style-type: none"> - Goal is to reach technical ability, but at the same time allow students to understand why a certain mathematical process is followed - Heavy emphasis on the technical aspects - Problems aren't very practical 	<ul style="list-style-type: none"> - Good environment - Teacher has control of the classroom - Teacher and students share the same values and respect the same rules

Table A4. Educator Groups' Impressions of the Hong Kong SAR Lessons

Educator Group	Teacher's role	Students' role	Content	Climate
Australia	<ul style="list-style-type: none"> Teacher is dominant 	<ul style="list-style-type: none"> Student involvement not highly valued 	<ul style="list-style-type: none"> Well structured Sequencing is important Textbook provides much of the learning structure Encourages mathematical thinking and reasoning Stress on relational understanding as opposed to rote understanding Appropriate technical language 	<ul style="list-style-type: none"> Students quiet and submissive The work is serious and important Little social interaction between teacher-students and students-students
Czech Republic	<ul style="list-style-type: none"> Teacher directed ("frontal") Lecture-like 	<ul style="list-style-type: none"> Minimal activity 	<ul style="list-style-type: none"> Formally structured Variety 	<ul style="list-style-type: none"> Formal and authoritative Great distance between students and teacher Based on "Asian discipline"
Hong Kong SAR	<ul style="list-style-type: none"> Teacher-led Knowledge is given by the teacher Well planned 	<ul style="list-style-type: none"> Students seem to be treated as calculation and computation machines A lot of student presentation No collaborative work Moderate mathematics demand placed on students 	<ul style="list-style-type: none"> Rich content Computation/calculation emphasized Rule-oriented Low emphasis on concepts Teacher mathematically competent 	<ul style="list-style-type: none"> Clear classroom routines
North America	<ul style="list-style-type: none"> Teacher dominated 	<ul style="list-style-type: none"> Moderate math demand on students 	<ul style="list-style-type: none"> Very textbook driven, based on examples worked out in the students' book 	<p><i>Opinions across the lessons were mixed</i></p>
Swiss-French	<ul style="list-style-type: none"> Teacher is authority Activities guided by the teacher 	<ul style="list-style-type: none"> Students given responsibility without autonomy (i.e. rarely ask questions, must do what 	<ul style="list-style-type: none"> High level of difficulty 	<ul style="list-style-type: none"> Students highly disciplined Students put forth a high degree of concentration and

Educator Group	Teacher's role	Students' role	Content	Climate
Swiss-German	<ul style="list-style-type: none"> - Teacher walks through step by step in a clear and direct way - Teacher shows the correct solution or solution method in response to mistakes - Teacher often answers their own questions 	<ul style="list-style-type: none"> - is proposed) - Often work out problems at the board that are judged by the teacher and peers - No independent cognitive constructing of the math structural knowledge - Students often answer teachers' questions together in sync ("choir style") - Participation limited to short, predictable statements 	<ul style="list-style-type: none"> - Opportunities for students to acquire a deeper understanding of math structures and principles - Training of procedural skills, with logical math explanations for the prescribed method - Incorrect student comments are eliminated through strong teacher guidance - Emphasis on correct use of math terminology - Following a review, new content is introduced and then practiced 	<p>effort, and have respect for the teacher</p> <ul style="list-style-type: none"> - Disciplined and work-intensive atmosphere - A high degree of pressure to succeed and a competitive character - Extrinsic incentives are emphasized
Swiss-Italian	<ul style="list-style-type: none"> - Repetition-imitative method - Seems to be a clear contract between the teacher and students 	<ul style="list-style-type: none"> - Very few instances when the students can actively participate - Students don't discover the learning process themselves - Goal is for students to gain instrumental competence and be competitive 	<ul style="list-style-type: none"> - Focused on competence in technical abilities - Lessons based on knowledge already known to students - No real "learning experiences" - Lesson objectives not clearly explained 	<ul style="list-style-type: none"> - Competitive - Use of microphone affects rapport between students and teacher

Table A5. Educator Groups' Impressions of the Dutch Lessons

Educator Group	Teacher's role	Students' role	Content	Climate
Australia	<ul style="list-style-type: none"> Not teacher dominated 	<ul style="list-style-type: none"> Little emphasis on public performance of students 	<ul style="list-style-type: none"> Sequence and structure of the math not emphasized Textbook provides much of the learning structure 	<ul style="list-style-type: none"> Casual relationships between teacher and students Many social interactions
Czech Republic	<ul style="list-style-type: none"> Teacher directed ("frontal") 	<ul style="list-style-type: none"> Much student responsibility Active Some use of groups 	<ul style="list-style-type: none"> Not very demanding Structured scheme: homework, new material, practice, then test 	<ul style="list-style-type: none"> Equality and partner-like relationship between teacher and students Mutual tolerance Liberal Informal
Hong Kong SAR	<ul style="list-style-type: none"> Low requirement on the teacher Teacher plays a "friendly" role 	<ul style="list-style-type: none"> No student presentation No collaborative work Students willing to ask questions Low expectations of students 	<ul style="list-style-type: none"> Superficial (shallow) math content Lesson covers a lot of questions (math problems), but there are no mathematics concepts involved 	<ul style="list-style-type: none"> Laissez-faire atmosphere Lack of discipline Class out of the teacher's control
North America	<ul style="list-style-type: none"> Teacher does not go farther than the textbook: "If the book does not ask, the teacher does not ask" 	<ul style="list-style-type: none"> Student's task is to work on assigned textbook problems independently High math demand on students 	<ul style="list-style-type: none"> Challenging topics and problems 	<p><i>Opinions across the lessons were mixed</i></p>
Swiss-French	<ul style="list-style-type: none"> Teacher guides and simplifies the work for students 	<p><i>Opinions across the lessons were mixed</i></p>	<ul style="list-style-type: none"> Focus is on rules and results Opportunities for math reasoning are not well exploited 	<p><i>Opinions across the lessons were mixed</i></p>

WHAT IS EFFECTIVE MATHEMATICS TEACHING?

Educator Group	Teacher's role	Students' role	Content	Climate
Swiss-German	<ul style="list-style-type: none"> - Teacher is coach, providing individual learning support - Teacher plays the role of facilitator - Lessons are guided only to a small degree by the teacher 	<ul style="list-style-type: none"> - Students have a high degree of control to explore procedures and construct their own math understanding - Mistakes are recognized as learning opportunities - Can get help from their classmates and tutoring from the teacher 	<ul style="list-style-type: none"> - Learning to solve problems by exploring, without instruction on standard solution procedures - Opportunities for students to acquire a deeper understanding of math structures and principles - Development of higher order thinking not observed - Relevant to real-life, and not very abstract - Little observable emphasis on correct mathematical language - Math terminology is casual and undemanding - Extensive discussion of prior homework, then new problems assigned and worked on independently for much of the lesson 	<ul style="list-style-type: none"> - Noise level is high, and discipline of students is a frequent issue - Chaotic - Very permissive and relaxed environment - No extrinsic motivators present
Swiss-Italian	<p><i>Opinions across the lessons were mixed</i></p>	<ul style="list-style-type: none"> - Students not involved, either due to teacher's lack of attempts or difficulty in finding the appropriate strategies 	<ul style="list-style-type: none"> - A division exists between theory and practice - No real occasions to learn 	<ul style="list-style-type: none"> - Teacher seems to be detached or has difficulty relating, and students seem distracted and uninterested - Efforts between the teacher and students to interact fail - Teacher and students lack rapport

Table A6. Educator Groups' Impressions of the Swiss Lessons

Educator Group	Teacher's role	Students' role	Content	Climate
Australia	<ul style="list-style-type: none"> Teacher is dominant 	<ul style="list-style-type: none"> Student activity and involvement valued 	<ul style="list-style-type: none"> Sequence and structure of the math emphasized Math thinking and reasoning encouraged Appropriate technical language 	<p><i>Opinions across the lessons were mixed</i></p>
Czech Republic	<ul style="list-style-type: none"> Teacher works together with students Well prepared 	<ul style="list-style-type: none"> Active in problem solving Focused, disciplined, motivated 	<ul style="list-style-type: none"> Structure focuses on problem solving Not very demanding, but well connected 	<ul style="list-style-type: none"> Good working morale and discipline Friendly relationship between teacher and students
Hong Kong SAR	<ul style="list-style-type: none"> Teacher-led Well planned 	<ul style="list-style-type: none"> High student engagement and involvement A lot of oral student presentation Individual student care, but no collaborative work A lot of student choice of mathematics/activities Quite high demand on students' work 	<ul style="list-style-type: none"> Rich math content Thinking process emphasized Calculation not emphasized 	<ul style="list-style-type: none"> Enjoyable atmosphere for students, self-motivation is quite high
North America	<i>Opinions across the lessons were mixed</i>	<ul style="list-style-type: none"> Degree of thinking students asked to do varies Math demand on students varies from low to moderate 	<i>Opinions across the lessons were mixed</i>	<i>No comments made</i>
Swiss-French	<ul style="list-style-type: none"> Teacher may provide indirect or subtle guidance 	<ul style="list-style-type: none"> Students are involved and participate in doing the work 	<i>Opinions across the lessons were mixed</i>	<i>No comments made</i>

Educator Group	Teacher's role	Students' role	Content	Climate
Swiss-German	<ul style="list-style-type: none"> Teacher prepares and highly structures the lessons based on didactics Knowledge is often developed in a teacher-led instructional conversation During individual work, teacher tries to diagnose mistakes and provide hints 	<ul style="list-style-type: none"> Students participate actively Some student directed learning situations, where they construct cognitive steps independently Discovery and individual (or group) problem-solving Mistakes are recognized as learning opportunities 	<ul style="list-style-type: none"> Focus is on understanding and meaning, rather than procedures Students have opportunities to acquire a deeper understanding of math structures and principles 	<ul style="list-style-type: none"> Respectful climate Lack of focus on extrinsic motivators
Swiss-Italian	<p><i>Opinions across the lessons were mixed</i></p>	<ul style="list-style-type: none"> Student's role varies from repeating procedures established by the teacher to time for "thought-reflection" and discovery 	<ul style="list-style-type: none"> The degree to which the lessons involve "learning situations" varies The educational objective of the lesson is not always clearly defined 	<ul style="list-style-type: none"> Quiet, serene, responsible environment

Table A7. Educator Groups' Impressions of the U.S. Lessons

Educator Group	Teacher's role	Students' role	Content	Climate
Australia	<ul style="list-style-type: none"> Teacher is dominant 	<ul style="list-style-type: none"> Student involvement not highly valued Little emphasis on public performance of students 	<ul style="list-style-type: none"> Reasoning not directly encouraged Stress on rote understanding as opposed to relational understanding Technical language overused 	<ul style="list-style-type: none"> Easy relationship between teacher and students Teachers accept a low level of student attention
Czech Republic	<ul style="list-style-type: none"> Teacher-controlled 	<ul style="list-style-type: none"> Independent work under teacher's guidance Goal is to pass test 	<ul style="list-style-type: none"> Material presented schematically without emphasis on fundamentals 	<ul style="list-style-type: none"> Teacher is "natural authority" Firm discipline

Educator Group	Teacher's role	Students' role	Content	Climate
Hong Kong SAR	<ul style="list-style-type: none"> - Teacher-led 	<ul style="list-style-type: none"> - Students not expected to think deeply and given low-level tasks - A lot of student choice of activities 	<ul style="list-style-type: none"> - Some structure - Not very demanding - Superficial ("slim") and simple math content - Emphasis on computation and factual recall 	<ul style="list-style-type: none"> - Order kept
North America	<ul style="list-style-type: none"> - Teacher helps students prepare for tests and doesn't explore ideas beyond what happened in the book 	<ul style="list-style-type: none"> - Students not asked to reason - Students not asked to make personal records of their work and the work of the class - Low math demand on students 	<ul style="list-style-type: none"> - Lessons tightly linked to materials in the text or provided by the publisher 	<p><i>No comments made</i></p>
Swiss-French	<ul style="list-style-type: none"> - Step by step guidance 	<ul style="list-style-type: none"> - Students don't hesitate to ask questions, but exchanges are only at an elementary level 	<ul style="list-style-type: none"> - Focus is on knowing "how" - Little or no justification of techniques and reasoning - Prior homework is discussed, but they don't get into what the difficulties were 	<ul style="list-style-type: none"> - Good ambience and relative freedom of students
Swiss-German	<ul style="list-style-type: none"> - Teacher demonstrates and students practice - Teacher demonstrates the content quickly but also with redundancy - Teacher shows the correct solution or solution method in response to mistakes 	<ul style="list-style-type: none"> - Verbal participation is infrequent and often limited to fill-in-the-blank questions - Even during seatwork, students guided step-by-step by the teacher - No opportunity for group work observed 	<ul style="list-style-type: none"> - Emphasis on procedural skills, not meaning - Procedures are not connected and integrated into a coherent knowledge structure - Higher order mathematical principles are not taught - Problems solved in small steps according to prescribed 	<ul style="list-style-type: none"> - Respectful climate - Motivation is extrinsic – to do well on the test

WHAT IS EFFECTIVE MATHEMATICS TEACHING?

Educator Group	Teacher's role	Students' role	Content	Climate
Swiss-Italian	<i>Opinions across the lessons were mixed</i>	<ul style="list-style-type: none"> - Very few instances when students can actively participate - Students don't discover the learning process themselves 	<ul style="list-style-type: none"> - procedures and rules - General, "open" problems not posed - Correct math terminology valued highly - Learning opportunities vary from none to learning only technical aspects - Lessons based on knowledge already known to students - Lesson objectives not clearly explained 	<ul style="list-style-type: none"> - The living experience of the students appears to be positive