Effective Mathematics Interventions

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Research to Practice: Literature

- Skills in mathematics have been low as compared to other subject areas
- Low enrollment in more advanced mathematics classes
- Developmental delays
 - Flexibility in using numbers
 - Number sense
 - Mental number lines
 - Arithmetic combinations and equations of numbers
 - Decomposing numbers
- Lack conceptual, deep understanding of mathematics
- Lack procedural fluidity and strategy use

National Assessment of Educational Progress (2009)

- Assessment in 5 areas:
 - Number and Operations
 - Measurement
 - Geometry
 - Data Analysis
 - Algebra
- Scores in grade 4 have increased since 1990, but are not significantly different since 2007
 - 82% performing at the basic level
 - 39% performing at the proficient level
 - 6% performing at the advanced level

NMAP Recommendations

- Focused mathematics curriculum to meet the critical foundational needs for algebraic readiness
 - Fluency with whole number computations
 - Proficiency with Fractions
 - Aspects of Geometry and Measurement
- Core instruction is not enough!
 - Strategy instruction
 - Explicit and systematic instruction
 - Conceptual understanding





Mathematics Performance in Algebra Readiness

Percentages of Students who Scored "At or above Proficient" National Assessment of Educational Progress



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Mathematics Performance in Algebra Readiness

• TAKS, State Data: Mathematics: Grade 5

	April 2011^0		
-	Number	Percent Met Std	Percent
	Tested	Panel Rec.	Performance
Total Tested:	348,487	86%	41%
African-American:	43,352	75%	26%
Hispanic:	174,366	83%	35%
White:	110,964	92%	52%
Eco-Dis:	210,096	81%	32%
LEP:	48,312	74%	23%
Special Ed:	17,884	62%	17%

Mathematics Performance in Algebra Readiness

• TAKS, State Data: Mathematics: Grade 10

	April 2011^0		
	Number Tested	Percent Met Std	Percent Commended Performance
		Panel Rec.	
Total Tested:	305,827	74%	18%
African-American:	39,626	61%	7%
Hispanic:	140,744	70%	12%
White:	107,102	83%	26%
Eco-Dis:	152,651	66%	10%
LEP:	12,916	44%	4%
Special Ed:	15,964	29%	2%

Procedures & Features: Tier II

- Identify an instructional sequence
 - Foundational
 - Multiple opportunities to practice within the lessons
 - Teach specific strategies
 - Build procedural knowledge
 - Increase student engagement
- Identify and teach prerequisite knowledge to build
 - Mastery
 - Fluency
- Quick pace
 - Use of time to stay on-task
 - Behavior management
- Error correction and scaffolds

Procedures & Features: Tier II

- Opportunities to make, show, write number concepts
- Enhance core curriculum through problem solving
- Regular, consistent intervention
 - 4-5 days per week
 - 20-30 minutes
- Progress monitoring
 - Daily
 - Independent Practice
 - 1-2 minutes
 - Reflect material taught
 - Weekly/Bi-weekly
 - Aim Checks
 - Generalization



Components of Explicit, Systematic Instruction

- All aspects of instruction include
 - Script
 - Time
 - Quick pace
 - Mix of choral and independent responses
 - Teacher talk decreases throughout lesson
- Preview/Cumulative Review (2-3 minutes)
 - Sets the tone for lesson
 - Purpose of lesson
 - Allows time to remediate/review skills
 - Choral and individual responses
- Modeled Practice (2-3 minutes)
 - Teaches the skill explicitly
 - Safe...students practice alongside or immediately following teacher directions
 - Quick pace
 - Small steps

Components of Explicit Instruction

- Guided Practice (6-8 minutes)
 - Similar to Modeled Practice
 - Increase individual time for practice
 - Provide error correction/scaffolds
 - Transaction from concrete to pictorial and/or pictorial to abstract
 - Teaches how to complete the daily check
- Independent Practice (2-3 minutes)
 - Fluency

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- Immediate feedback
- Error correction
- Total Time: 12-17 minutes

Multiple Representations

- Concrete: Modeling/Guided Practice
 - Cubes
 - Counters
 - Base-ten/Place value materials (units, rods, flats)
 - Dot cubes
- Pictorial: Guided Practice/Independent Practice
 - Five frames
 - Ten frames
 - Hundreds charts
 - Number lines
- Abstract: Guided Practice/Independent Practice
 - Numbers
- Mats (can be used across all representations)
 - Part-part-whole
 - Fact family
 - Strategy mats



Student Verbalizations

- Student Verbalizations includes students' thinking aloud about their problem solving approaches, mathematical understanding, or promoting mathematical discourse (Gersten et al., 2009)
 - Questions
 - Neighbor-share
 - Choral response
 - Wipe boards
 - Multiple choice answers
 - Identifying mistakes



Student Verbalizations



Say: What fraction is shown here? $(\frac{8}{12})$ I want to show that $\frac{8}{12}$ is equivalent to $\frac{2}{3}$. How would I do that?

Listen for students to say, "Combine 4 smaller parts to make larger parts." As needed, give hints such as, "Remove division lines to make bigger parts."

Circulate to monitor student progress. Randomly stop, draw attention to a completed cell, and ask: How did you get this answer? If you know this fact, what other fact do you know? Why?

Example: Draw attention to the cell containing 9×12 on the Practice 21 handout. Listen for a student to explain that 9 can be taken apart into 4 + 5, so the products from 4×12 and 5×12 were added to solve 9×12 because the 9s facts can be found by taking apart the 9s facts into 4s facts plus the 5s facts. Listen for students to explain that if 9×12 is known, 12×9 is also known because of the commutative property of multiplication and watch for students to complete the row and column.



Visual Representations

• Using Visual Representations used during instruction include those used by the teacher to model problem solving as well as student use of manipulatives (Gersten et al., 2009).



Visual Representations

A guitar has 6 strings. Adam has 8 guitars. Draw a number line to show how many guitar strings there are in all.



There are <u>48</u> strings in all.

5 x 6

3 Tier Mathematics Model

- Free to Texas Educators at <u>http://3tiermathmodel.org/</u>
- Username: Texas Teacher
- Password: mathematics



Content and Skills: Grades K-2nd

- Word Problem Solving:
 - Strategy to solve all types
 - Different types of problems
 - Extraneous information
 - Multiple steps, contextualized
- Number Knowledge and Relationships
 - Counting: Rote, Rational, Counting Up/Back, Skip (2, 5, 10)
 - Number Recognition & Writing: 0-20 (kinder) 0 99 (1st); 0 999 (2nd grade)
- Number Relationships of greater than/less than/equal to
 - Relationships of one and two more than/less than
 - Anchoring Numbers to 5 & 10 frames
 - Part-part-whole Relationships (e.g., ways to represent numbers)
- Numeric Sequencing
 - Number line, mental number line
 - Math flexibility
 - Ordering numbers



Grades K-2nd (cont.)

- Base 10 & Place Value
 - Making and counting:
 - Groups of tens and ones (1st grade)
 - Groups of hundreds, tens, and ones (2nd grade)
 - Using base-ten language (3 hundreds, 0 tens, 6 ones) and standard language (306) to describe place value
 - Reading and writing numbers to represent base ten models
 - Naming the place value held by digits in numbers
- Addition & Subtraction Combinations
 - Identity Element and Properties
 - Fact Families
 - Counting & Decomposition Strategies
 - Addition: count on, [+ 0, + 1, + 2], doubles, doubles +1, make 10 + more
 - Subtraction: count down [-0, -1, -2, -3], count on

Intervention Modules

Grade 3

- Place Value Concepts
- Addition & Subtraction of Whole Numbers
- Multiplication & Division
 Concepts
- Fraction Concepts

Grade 4

- Multiplication & Division
 Strategies
- Multiplication & Division
 of Whole Numbers
- Modeling, Comparing, & Ordering Fractions
- Fraction & Decimal Relationships

Module A: Place Value Concepts

• The students is expected to:

- Use place value to read, write (in symbols and words) and describe the value of whole numbers through 999,999. (3.1 A, supporting) • Use place value to compare and order whole numbers through 9.999. (3.1 B, supporting)
- Identify and extend whole-number patterns. (3.6 A; supporting)
- Use data to describe events as more likely than, less likely than, or equally likely as. (3.13 C, supporting)
- Applies Grade 3 math to solve problems connected to everyday experiences in and outside of school. The student is expected to understand the problem, make a plan, carry out the plan and evaluate the solution for reasonableness. (3.14 A; B; C; D)
 Can relate informal language to mathematical language and symbols. (3.15 B)

Module B: Addition & Subtraction of Whole Numbers

• The students is expected to:

- Model addition and subtraction using pictures, words and numbers (3.3 A, supporting)
- Select addition or subtraction and use the operation to solve problems involving whole numbers through 999 (3.3 B, readiness)
 Round whole numbers to the nearest ten or hundred to approximate reasonable results in problem situations. (3.5 A, supporting)
- Use strategies including rounding and compatible numbers to estimate solutions to addition and subtraction problems. (3.5 B, supporting)
- Identify and extend whole-number patterns to make predictions and solve problems (3.6 A, supporting)
- Applies Grade 3 math to solve problems connected to everyday experiences in and outside of school. The student is expected to understand the problem, make a plan, carry out the plan and evaluate the solution for reasonableness. (3.14 A; B; C; D)
- Can relate informal language to mathematical language and symbols. (3.15 B)

Module C: Multiplication and Division Concepts

• The students is expected to:

- Learn and apply multiplication facts through 12 by 12 using concrete models and objects (3.4 A, supporting)
- Solve and record multiplication problems. (3.4 B, readiness)
- Use models to solve division problems and use number sentences to record the solutions. (3.4 C, supporting)
- Identify patterns in multiplication facts using concrete objects, pictorial models or technology. (3.6 B, supporting)
- Identify patterns in related multiplication and division sentences (fact families). (3.6 C, supporting)
- Applies Grade 3 math to solve problems connected to everyday experiences in and outside of school. The student is expected to understand the problem, make a plan, carry out the plan and evaluate the solution for reasonableness. (3.14 A; B; C; D)
- Can relate informal language to mathematical language and symbols. (3.15 B)

Module D: Fraction Concepts

• The students is expected to:

- Compare fractional parts of whole objects or sets of objects in a problem situation using concrete models (3.2 B, readiness)
- Use fraction names and symbols to describe fractional parts of whole objects or sets of objects (3.2 C, readiness)
- Construct concrete models of equivalent fractions for fractional parts of whole objects. (3.2 D)
- Student recognizes that a line can be used to represent numbers and fractions and their properties and relationships. Can locate and name points on number line using whole numbers and fractions, including halves and fourths. (3.10 A, readiness)
- Applies Grade 3 math to solve problems connected to everyday experiences in and outside of school. The student is expected to understand the problem, make a plan, carry out the plan and evaluate the solution for reasonableness. (3.14 A; B; C; D)
 Can relate informal language to mathematical language and symbols. (3.15 B)

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Module A: Multiplication & Division Strategies

The students is expected to:

- Use multiplication and division to solve problems without technology. (4.4 D, E, readiness)
- Model factors and products using arrays and area models. (4.4 A, supporting; 4.4 B, supporting)
- Represent multiplication and division situations in picture, word and number form. (4.4 B, supporting)
- Use multiplication and division to solve problems (4.4 D, E, readiness)
- Use patterns and relationships to develop strategies to remember basic multiplication and division facts (4.6 A, supporting)
- Represent multiplication and division situations in picture, word, and number form. (4.4 B, supporting)
- Recall and apply multiplication facts through 12 x 12. (4.4 C, supporting)
- Applies Grade 4 math to solve problems connected to everyday experiences in and outside of school. The student is expected to understand the problem, make a plan, carry out the plan
 and evaluate the solution for reasonableness. (4.14 A; B; C; D)
- Can relate informal language to mathematical language and symbols. (4.15 B)
- Make generalizations from patterns or sets of examples and nonexamples (4.16 A)
 Justify why an answer is reasonable and explain the solution process. (4.16 B)

Module B: Multiplication & Division of Whole Numbers

- The students is expected to:
- Use multiplication and division to solve problems (no more than 2 digits times 2 digits or one digit divisor and three-digit dividends without technology). (4.4 D; E, readiness)
 Model factors and products using arrays and area models (4.4 A, supporting)
- Represent multiplication and division situations in picture, word and number form. (4.4 B, supporting)
- Use strategies including rounding and compatible numbers to estimate solutions to multiplication and division problems (4.5 B, supporting)
- Use patterns to multiply by 10 and 100. (4.6 B, supporting)
- Applies Grade 4 math to solve problems connected to everyday experiences in and outside of school. The student is expected to understand the problem, make a plan, carry out the plan
 and evaluate the solution for reasonableness. (4.14 A; B; C; D)
- Can relate informal language to mathematical language and symbols. (4.15 B)
- Make generalizations from patterns or sets of examples and nonexamples (4.16 A)
- Justify why an answer is reasonable and explain the solution process. (4.16 B)

Module C: Modeling, Comparing & Ordering Fractions

The students is expected to:

- Model fraction quantities greater than one using concrete objects and pictorial models. (4.2 B, supporting)
- Use concrete objects and pictorial models to generate equivalent fractions. (4.2 A, supporting)
- To locate and name points on a number line using whole numbers and fractions such as halves and fourths. (4.10 A, readiness)
- Compare and order fractions using concrete objects and pictorial models. (4.2 C, supporting)
- Applies Grade 4 math to solve problems connected to everyday experiences in and outside of school. The student is expected to understand the problem, make a plan, carry out the plan and evaluate the solution for reasonableness. (4.14 A; B; C; D)
- Can relate informal language to mathematical language and symbols. (4.15 B)
- Make generalizations from patterns or sets of examples and nonexamples (4.16 A)
- Justify why an answer is reasonable and explain the solution process. (4.16 B)

Module D: Fraction & Decimal Relationships

- The students is expected to:
- Relate decimals to fractions that name tenths and hundredths using concrete objects and pictorial models. (4.2 D, readiness)
- Locate and name points on a number line using whole numbers, fractions such as halves and fourths, and decimals such as tenths. (4.10 A, readiness)
- Relate decimals to fractions that name tenths and hundredths using concrete objects and pictorial models. (4.2 D, readiness)
- Use place value to read, write, compare and order decimals involving tens and hundredths, including money, using concrete objects and pictorial models. (4.1 B, readiness; 4.2 C, supporting)
- Applies Grade 4 math to solve problems connected to everyday experiences in and outside of school. The student is expected to understand the problem, make a plan, carry out the plan
 and evaluate the solution for reasonableness. (4.14 A; B; C; D)
- Can relate informal language to mathematical language and symbols. (4.15 B)
- Make generalizations from patterns or sets of examples and nonexamples (4.16 A)
- Justify why an answer is reasonable and explain the solution process. (4.16 B)

Intervention 6th – 9th Grade

- Project Share
 - MSTAR: Middle School Intervention
 - Facts & Patterns in Multiplication and Division
 - Proportionality
 - Ratios and Rates
 - Equivalent Fractions
 - H.S. Algebra Intervention Modules
 - Variables
 - Expressions, Equations, & Equivalence



Students Who Struggle....

Table 2: Benchmarks for the Critical Foundations

Fluency With Whole Numbers

- 1) By the end of Grade 3, students should be proficient with the addition and subtraction of whole numbers.
- 2) By the end of Grade 5, students should be proficient with multiplication and division of whole numbers.

Fluency With Fractions

- By the end of Grade 4, students should be able to identify and represent fractions and decimals, and compare them on a number line or with other common representations of fractions and decimals.
- 2) By the end of Grade 5, students should be proficient with comparing fractions and decimals and common percent, and with the addition and subtraction of fractions and decimals.
- By the end of Grade 6, students should be proficient with multiplication and division of fractions and decimals.
- 4) By the end of Grade 6, students should be proficient with all operations involving positive and negative integers.
- 5) By the end of Grade 7, students should be proficient with all operations involving positive and negative fractions.
- 6) By the end of Grade 7, students should be able to solve problems involving percent, ratio, and rate and extend this work to proportionality.

Geometry and Measurement

- By the end of Grade 5, students should be able to solve problems involving perimeter and area of triangles and all quadrilaterals having at least one pair of parallel sides (i.e., trapezoids).
- 2) By the end of Grade 6, students should be able to analyze the properties of two-dimensional shapes and solve problems involving perimeter and area, and analyze the properties of threedimensional shapes and solve problems involving surface area and volume.
- By the end of Grade 7, students should be familiar with the relationship between similar triangles and the concept of the slope of a line.

The Major Topics of School Algebra

Symbols and Expressions

- Polynomial expressions
- Rational expressions
- Arithmetic and finite geometric series

Linear Equations

- · Real numbers as points on the number line
- · Linear equations and their graphs
- Solving problems with linear equations
- Linear inequalities and their graphs
- Graphing and solving systems of simultaneous linear equations

Quadratic Equations

- · Factors and factoring of quadratic polynomials with integer coefficients
- Completing the square in quadratic expressions
- Quadratic formula and factoring of general quadratic polynomials
- Using the quadratic formula to solve equations

Functions

- Linear functions
- · Quadratic functions—word problems involving quadratic functions
- · Graphs of quadratic functions and completing the square
- Polynomial functions (including graphs of basic functions)
- Simple nonlinear functions (e.g., square and cube root functions; absolute value; rational functions; step functions)
- Rational exponents, radical expressions, and exponential functions
- Logarithmic functions
- Trigonometric functions
- Fitting simple mathematical models to data

Algebra of Polynomials

- Roots and factorization of polynomials
- Complex numbers and operations
- Fundamental theorem of algebra
- Binomial coefficients (and Pascal's Triangle)
- Mathematical induction and the binomial theorem

Combinatorics and Finite Probability

 Combinations and permutations, as applications of the binomial theorem and Pascal's Triangle

~ Taken from National Mathematics Advisory Panel, 2008, p. 16, 20

