CLOSING THE ACHIEVEMENT GAP

Best Practices In Teaching Mathematics

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Introduction

Mathematics is a form of reasoning. Thinking mathematically consists of thinking in a logical manner, formulating and testing conjectures, making sense of things, and forming and justifying judgments, inferences, and conclusions. We demonstrate mathematical behavior when we recognize and describe patterns, construct physical and conceptual models of phenomena, create symbol systems to help us represent, manipulate, and reflect on ideas, and invent procedures to solve problems (Battista, 1999).

Recent national test results provide continuing documentation of the need to increase the focus on improving student achievement in mathematics. The National Assessment of Educational Progress (NAEP) recently released the 2005 math scores which reflected student achievement in the areas of measurement, geometry, data analysis, probability and algebra. Nationally, only 30% of eighth graders were deemed proficient. Although reflecting an increase from previous assessments, only 69% of the eighth graders nationally demonstrated a basic skills level on the NAEP assessment (Olson, 2005).

The need for effective instruction in mathematics was further documented in a February 2006 study by the U.S. Department of Education. The study findings are based on data from a nationally representative sample of students from the high school class of 1992 who attended a four-year college. The study found that taking a full schedule of academically demanding courses in high school, including mathematics beyond

Algebra II, was the single most significant pre-collegiate variable in determining if students graduated from college. The study also found significant disconnects between the high school curriculum and the expectations of the first year of college, suggesting the need to increase the level of challenging academic content in high school. This need to offer a more challenging high school curriculum is even more critical for poor and minority students as they are less likely than higher socioeconomic and white students to attend high schools that offer a challenging curriculum. States moving to increase unit requirements for graduation must also attend to content requirements if they expect to make a difference in student performance (Adelman, 2006).

Masini and Taylor (2000) report research documenting that the number of mathematics topics covered prior to eighth grade is positively correlated to mathematics achievement while the number of new topics presented at the eighth grade level is negatively correlated to mathematics achievement. Regardless of math skills before high school, taking algebra in the middle school is strongly related to achievement gains in high school. The math curriculum must provide students with opportunities to learn math at an early age.

The poor performance of U.S. students in math can be traced to the method used to teach math at the elementary level. The focus is on specific problems and not on building the foundations necessary for understanding higher level math. These foundations can only be built with a mathematics program that teaches concepts and skills, and problem-solving (Daro, 2006).

The Mathematics Reform Movement

The reform movement in mathematics education can be traced to the mid-1980's and was a response to the failure of traditional teaching methods, the impact of technology on curriculum and the emergence of new approaches to the scientific study of how mathematics is learned. Basic to the reform movement was a standards-based approach to the "what and how" of mathematics teaching (Battista, 1999).

In the new mathematics, the focus is on problem solving, mathematical reasoning, justifying ideas, making sense of complex situations and independently learning new ideas. Students must be provided with opportunities to solve complex problems, formulate and test mathematical ideas and draw conclusions. Students must be able read, write and discuss mathematics, use demonstrations, drawings and real-world objects, and participate in formal mathematical and logical arguments (Battista, 1999).

The driving force behind the standards-based approach to mathematics instruction has been the standards developed by the National Council of Teachers of Mathematics (NCTM). The *Principles and Standards for School Mathematics*, published by NCTM in 2000, outlines the principles and standards for developing a comprehensive school mathematics program. The document delineates six guiding principles related to equity, curriculum, teaching, learning, assessment and technology, and identifies five content and process standards outlining what content and processes students should know and be able to use. The content standards are organized around content strands related to numbers and operations, algebra, geometry, measurement and data analysis and probability. The process standards are organized around the areas of problem solving, reasoning and proof, communication, connections and representations (National Council of Teachers of Mathematics, 2000).

A set of basic assumptions about teaching and schooling practices is implicit in this reform movement. First, all students must have an opportunity to learn new mathematics. Second, all students have the capacity to learn more mathematics than we have traditionally assumed. Third, new applications and changes in technology have changed the instructional importance of some mathematics concepts. Fourth, new instructional environments can be created through the use of technological tools. Fifth, meaningful mathematics learning is a product of purposeful engagement and interaction which builds on prior experience (Romberg, 2000).

A recent concept paper published by the American Mathematical Society has been influential in identifying some common areas of agreement about mathematics education. The identified areas of agreement are based on three fundamental premises; basic skills with numbers continue to be important and students need proficiency with computational procedures, mathematics requires careful reasoning about precisely defined objects and concepts, and students must be able to formulate and solve problems. The areas of agreement emerging from these premises include:

- Mathematical fluency requires automatic recall of certain procedures and algorithms.
- Use of calculators in instruction can be useful but must not impede the development of fluency with computational procedures and basic facts.
- Using and understanding the basic algorithms of whole number arithmetic is essential.

- Developing an understanding of the number meaning of fractions is essential.
- Teachers must ensure that the use of "real-world" contexts for teaching mathematics maintains a focus on mathematical ideas.
- Mathematics should be taught using multiple strategies, however, the teacher is responsible for selecting the strategies appropriate for a specific concept.
- Mathematics teachers must understand the underlying meaning and justifications for ideas and be able to make connections among topics.

(Ball, Ferrini-Mundy, Kilpatrick, Milgram, Schmid, & Scharr, 2005).

Standards-Based Mathematics

Standards-based instruction in mathematics is designed to clearly identify what students should learn at each level. Standards provide more than a curriculum framework as they delineate the skills, concepts and knowledge that are to be mastered. For successful standards-based implementation, teachers must understand the rationale for using standards, know applicable national and state standards and use them as a basis for planning instruction, and implement best practices instructional strategies. Essential characteristics of an effective standards-based mathematics classroom include:

- Lessons designed to address specific standards-based concepts or skills.
- Student centered learning activities.
- Inquiry and problem solving focused lessons.
- Critical thinking and knowledge application skills
- Adequate time, space, and materials to complete tasks.
- Varied, continuous assessment, designed to evaluate both student progress and teacher effectiveness. (Teaching Today, 2005a)

The implementation of a standards-based math curriculum brings with it some special challenges. In addition to ensuring students are actively engaged, teachers should adhere to the following guidelines:

- Create a safe environment where students feel comfortable.
- Establish clear procedures and routines.
- Provide both challenge and support.
- Use carefully assigned and well-managed cooperative groups.
- Make frequent real life connections.
- Use an integrated curriculum.
- Provide engaging educational experiences that are relevant to students.
- Present activities where students produce and share products.

(Teaching Today, 2005b, ¶ 3)

The Thomas B. Fordham Foundation has conducted three analyses of state mathematics standards. The most recent study was released in 2005. Although the weighting of the specific criteria has shifted, the same criteria: clarity of the standards, content, sound mathematical reasoning, and the absence of negative features, have been used to evaluate standards in each of the studies. Overall, only six states received grades of A of B. Twenty-nine states received grades of D or F, and 15 received Cs. The report identified nine major areas of concern including excessive emphasis on calculator use, memorization of basic number facts, lack of focus on the standard algorithms, insufficient focus on fractions, inadequate attention to mathematical patterns, counterproductive use of manipulatives, overemphasis on estimation skills, improper sequencing of statistics and probability standards, and a lack of standards that appropriately guide the development of problem-solving (Klein, 2005).

The study also offers suggestions for state policy makers seeking to strengthen their K-12 math standards. These recommendations include the use of standards developers who thoroughly understand mathematics, the development of coherent arithmetic standards that emphasize both conceptual understanding and computational fluency, avoid and rectify the nine major concerns related to math standards, and consider borrowing a complete set of math standards from one of the states with high-quality standards (Klein, 2005).

Best Practices

Sabean and Bavaria (2005) have synthesized a list of the most significant principles related to mathematics teaching and learning. This list includes the expectations that teachers know what students need to learn based on what they know, teachers ask questions focused on developing conceptual understanding, experiences and prior knowledge provide the basis for learning mathematics with understanding, students provide written justification for problem solving strategies, problem based activities focus on concepts and skills, and that the mathematics curriculum emphasizes conceptual understanding.

Concurrently, the following best practices for implementing effective standards-based math lessons should be followed:

- Students' engagement is at a high level.
- Tasks are built on students' prior knowledge.

- Scaffolding takes place, making connections to concepts, procedures, and understanding.
- High-level performance is modeled.
- Students are expected to explain thinking and meaning.
- Students self-monitor their progress.
- Appropriate amount of time is devoted to tasks.

(Teaching Today, 2005b, ¶ 7)

The role of discovery and practice and the use of concrete materials are two additional topics that must be considered when developing a program directed at improving mathematics achievement. Sabean and Bavaria (2005) examined research which suggested that such a program must be balanced between the practice of skills and methods previously learned and new concept discovery. This discovery of new concepts, they suggest, facilitates a deeper understanding of mathematical connections.

Johnson (2000) reported findings that suggest that when applied appropriately, the long-term use of manipulatives appears to increase mathematics achievement and improve student attitudes toward mathematics. The utilization of manipulative materials helps students understand mathematical concepts and processes, increases thinking flexibility, provides tools for problem-solving, and can reduce math anxiety for some students. Teachers using manipulatives must intervene frequently to ensure a focus on the underlying mathematical ideas, must account for the "contextual distance" between the manipulative being used and the concept being taught, and take care not to overestimate the instructional impact of their use.

Sabean and Bavaria (2005) have summarized research suggesting that the development of practical meaning for mathematical concepts is enhanced through the use of manipulatives. They further suggest that the use of manipulatives must be long term and meaningfully focused on mathematical concepts.

The National Council of Teachers of Mathematics has developed a position statement which provides a framework for the use of technology in mathematics teaching and learning. The NCTM statement endorses technology as an essential tool for effective mathematics learning. Using technology appropriately can extend both the scope of content and range of problem situations available to students. NCTM recommends that students and teachers have access to a variety of instructional technology tools, teachers be provided with appropriate professional development, the use of instructional technology be integrated across all curricula and courses, and that teachers make informed decisions about the use of technology in mathematics instruction (National Council of Teachers of Mathematics, 2003).

Acknowledging and responding to the varied learning styles of students is a critical component of effective inquiry oriented standards-based math instruction. Effective strategies for differentiating mathematics instruction include rotating strategies to appeal to students' dominant learning styles, flexible grouping, individualizing instruction for struggling learners, compacting (giving credit for prior knowledge), tiered assignments, independent projects, and adjusting question level (Computing Technology for Math Excellence, 2006).

A 1998 meta-analysis of 100 research studies on teaching mathematics provided support for a three-phase instructional model. In the first phase of the model, teachers demonstrated, explained, questioned, conducted discussions and checked for understanding. Students are actively involved in discussions and responding to questions. In phase two, teachers and student peers provide student assistance that is gradually reduced while students receive feedback on their performance, corrections, additional explanations, and other assistance as needed. In phase three, teachers assess students' ability to apply the knowledge gained while students demonstrate their ability to recall, generalize or transfer what they have learned. Effective lessons do not require students to apply new knowledge independently until they have demonstrated an ability to successfully do so (Dixon, Carnine, Lee, Wallin, & Chard, 1998).

The recent results from the Third International Mathematics and Science Study (TIMSS) have caused many teachers in the United States and Canada to take a closer look at strategies and techniques used by Japanese teachers in teaching mathematics. TIMSS results documented the advanced performance and more in depth mathematical thinking of Japanese students. A central strategy in the success of the Japanese mathematics teachers has been the use of Lesson Study, an instructional approach that includes a group of teachers developing, observing, analyzing and revising lesson plans that are focused on a common goal. This process is focused on improving student thinking and includes selecting a research theme, focusing the research, creating the lesson, teaching and observing the lesson, discussing the lesson, revising the lesson and documenting the findings. A key element of the Lesson Study process is that it helps to facilitate teachers working together using interconnecting skills across grade levels and lessons (Teaching Today, 2006).

Professional Development, High Quality Teaching and Student Achievement in Mathematics

A September, 2005 report, *A Study of Professional Development for Public School Educators in West Virginia*, provides a framework for viewing the relationship between professional development, teacher quality, and student achievement in mathematics. The report, prepared by the National Staff Development Council for the Legislative Oversight Commission on Education Accountability, notes that there is conclusive evidence from current research that the single most critical factor in improving student learning is teaching quality. Concurrently, the accountability provisions of NCLB have substantially increased the pressure on states and school districts to provide highly qualified teachers. A number of legislative and policy initiatives have been implemented to ensure that teachers entering the classroom are high quality. Experienced teachers, however, must look to professional development to expand their content knowledge and assist in learning new standards-based instructional strategies (National Staff Development Council, 2005).

The training and preparation received by many current teachers did not prepare them to address the new student performance standards which stress higher-order thinking and analytical skills and require teachers to teach the use of critical thinking, problem solving and inquiry. Teachers are not able to teach what they do not know. Consequently, the role of professional development in assuring quality teaching for experienced teachers is critical (National Staff Development Council, 2005).

Using the concept of professional development and professional learning interchangeably, the No Child Left Behind Act provides the following definition of high-quality professional development:

> Professional learning needs to give teachers and school leaders the skills to support students' mastery of states' academic standards; enhance the content knowledge of teachers in their teaching subjects; be integrated into overall school and district improvement plans; be research based; align with state student standards; and be sustained, intensive, and focused on classroom practice. In fact, the legislation explicitly specifies that one-day or short-term workshops do not qualify as effective professional development. (U.S. Department of Education, 2002, p.13)

This definition is closely aligned with the professional development standards developed by the National Staff Development Council (2001).

A recent research report, *The Role of Professional Development for Teachers* (2005), published by the Education Alliance, provides additional support for the use of the NCLB definition and NSDC standards. Synthesizing the current research on effective professional development, the report concludes that effective professional development for teachers is teacher driven, ongoing and sustained, school-based and job-embedded, content-focused, focused on student needs and uses appropriate adult learning strategies.

Administrators in Boston and San Diego believe that a concentrated focus on building students' conceptual math skills and investing in professional development for their elementary and middle school teachers were major factors behind their gains on the most recent National Assessment of Educational Progress. Both school systems worked to lessen the focus on memorizing facts, formulas and procedures, and increase the emphasis on developing problem solving skills. The districts also provided additional support for teachers by providing additional instructional materials such as curriculum maps and pacing guides. Other support strategies include the establishment of math leadership teams and providing math coaches (Cavanagh, 2006).

Mathematics Instruction and Assessment

Johnson (2000) reported research suggesting that the impact of standards in establishing external assessment expectations is profound. Understanding these standards and their related assessments allows teachers to plan and adjust instruction accordingly. Effective assessment of mathematics learning must be performance-based, use multiple strategies and employ more open-ended assessment tasks than have been used in the past.

Effective assessment practices are essential to support mathematics instruction that produces improved student performance. Teachers and students have been placed under tremendous pressure to prepare students for the accountability measures and standardized tests required by the No Child Left Behind legislation. Despite these pressures, mathematics teachers must resist the tendency to rely on the results of standardized tests only to measure student performance in mathematics (Computing Technology for Math Excellence, 2006).

Assessment in a standards-based environment requires that students be judged in terms of mathematical literacy, understanding of concepts and procedures, and the application of mathematical knowledge in problem-solving situations. Since most traditional assessment strategies were not designed for these purposes, new assessment models must be developed. One such model, developed by the Organization for Economic Cooperation and Development, focuses on assessing large ideas such as change and growth, space and shape, and chance. The model also organizes the assessment of thinking skills into three categories focused on tasks requiring simple computations and definitions, tasks requiring that connections be made to solve problems and tasks requiring higher level mathematical thinking and analysis (Romberg, 2000).

Assessment strategies can be classified as diagnostic, formative or summative. The manner in which teachers use assessment in their instruction is a major variable in determining student achievement. Diagnostic assessment strategies are focused on assessing students' prior knowledge, strengths, weaknesses and skill levels. Formative assessments are directed at providing immediate feedback and evidence of student performance. Summative assessments are more comprehensive and are typically administered at the end of a specific unit or timeframe (Computing Technology for Math Excellence, 2006).

Assessment strategies can also be characterized as traditional or alternative in nature. Multiple choice, true/false or matching tests represent traditional approaches to assessment, whereas, strategies such as portfolios, journal writing, student self-assessment, and performance tools may be considered alternative assessment strategies. Traditional and alternative assessments may be used for diagnostic, formative or summative purposes (Computing Technology for Math Excellence, 2006).

Best Practices Summary

Viewed from the classroom, mathematics instruction that is standards-based is different from traditional mathematics instruction. Students approach mathematics differently as they explore functions, develop formulas, and actively engage in nonroutine problem-solving and interaction about mathematics. Students use calculators for computational assistance and as tools for solving problems. This open and focused approach on problem-solving, reasoning and communication processes allows teachers and students to learn from each other (Smith, Smith, & Romberg, 1993).

Research from the past 15 years provides a clear picture of the impact of a standards-based math curriculum. Students who take rigorous mathematics courses are much more likely to go to college than those who do not. The gateway to advanced mathematics in high school is Algebra. We also know that achievement in mathematics is based on the type of courses a student takes, not the type of school attended (U.S. Department of Education, 1997).

As evidenced in this brief review of the literature related to teaching mathematics, there is a literature basis for a set of best practices for use in teaching mathematics. These recommended practices are summarized in the following chart.

Instructional Element	Recommended Practices
Curriculum Design	• Ensure mathematics curriculum is based on challenging content • Ensure curriculum is standards-based • Clearly identify skills, concepts and knowledge to be mastered • Ensure that the mathematics curriculum is vertically and horizontally articulated
Professional Development for Teachers	• Provide professional development which focuses on: ▪ Knowing/understanding standards ▪ Using standards as a basis for instructional planning ▪ Teaching using best practices ▪ Multiple approaches to assessment • Develop/provide instructional support materials such as curriculum maps and pacing guides • Establish math leadership teams

	and provide math coaches
Technology	• Provide professional development on the use of instructional technology tools • Provide student access to a variety of technology tools • Integrate the use of technology across all mathematics curricula and courses

Manipulatives	• Use manipulatives to develop understanding of mathematical concepts • Use manipulatives to demonstrate word problems • Ensure use of manipulatives is aligned with underlying math concepts
Instructional Strategies	• Focus lessons on specific concept/skills that are standards-based • Differentiate instruction through flexible grouping, individualizing lessons, compacting, using tiered assignments, and varying question levels. • Ensure that instructional activities are learner-centered and emphasize inquiry/problem-solving • Use experience and prior knowledge as a basis for building new knowledge • Use cooperative learning strategies and make real life connections • Use scaffolding to make connections to concepts, procedures and understanding • Ask probing questions which require students to justify their responses • Emphasize the development of basic computational skills
Assessment	• Ensure assessment strategies are aligned with standards/concepts being taught • Evaluate both student progress/performance and teacher effectiveness • Utilize student self-monitoring techniques • Provide guided practice with feedback • Conduct error analyses of student work

	• Utilize both traditional and alternative assessment strategies • Ensure the inclusion of diagnostic, formative and summative strategies • Increase use of open-ended assessment techniques

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